Quantum Teleportation

Protocol for transferring qubit from one party to another
Perfectly preserves state information

\[ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \]

\[ |\psi'\rangle = \alpha|0\rangle + \beta|1\rangle \equiv |\psi\rangle \]

Alice to Bob

Quantum Teleportation allows for transmission over great distances...think from Earth to space!
Not an exaggeration: In 2017, qubits were teleported from Earth to space!

Requirements for Quantum Teleportation

Alice has a 'message' qubit \( |\psi\rangle \) that she wants to send to Bob

Teleportation protocol requires additional resources:
- Two entangled qubits...each party has one half of the entangled pair
- A classical communication line used to send two classical bits from Alice to Bob

Note: Since quantum teleportation needs classical communication, it cannot be faster than the speed of light!
Step 1: Create an entangled pair of qubits

Entangle two qubits using the same entangle without phase.
- Alice and Bob can entangle the qubits, or they can receive them from a third party!

\[ |\psi_{in}\rangle = |00\rangle \]
\[ |\psi_{out}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \]

Step 2: Distribute entangled qubits to Alice & Bob

Alice now has two qubits:
- Half of the entangled pair
- The message qubit, \(|\psi\rangle\)

Calculating total quantum state:
\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

State 3

State 2

\[ \frac{1}{2} [\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle)] + \frac{1}{2} [\beta(|000\rangle + |011\rangle + |100\rangle + |111\rangle)] \]

Recall that Alice has the first 2 qubits, Bob has the last qubit.

Step 3: Alice applies CNOT to her qubits

Alice applies CNOT on:
- Message qubit, \(|\psi\rangle\) (control)
- Her entangled qubit (target)

Group terms suggestively:
\[ \frac{1}{2} [\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle)] + \frac{1}{2} [\beta(|000\rangle + |011\rangle + |100\rangle + |111\rangle)] \]

Step 4: Alice applies an H gate on \(|\psi\rangle\)

Three-Qubit State Transformation

\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

State 3

State 2

Group terms suggestively:
\[ \frac{1}{2} [\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle)] + \frac{1}{2} [\beta(|000\rangle + |011\rangle + |100\rangle + |111\rangle)] \]
Step 5: Alice measures both of her qubits

Step 6: Process results of measurement

We deduce information about Bob’s state by using partial measurements.

Step 7: Alice transmits her two classical bits to Bob

Step 8: Bob recovers $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$

To recover $|\psi\rangle$:
- If $b_0$ is 1, then apply a Z gate
- If $b_0$ is 0, then apply a NOT (X) gate
Full Quantum Circuit for Teleportation of $|\psi\rangle$

**Practice:**
During teleportation, Alice measures her qubits and sends Bob the bit string $b_1b_0 = 10$. If the message qubit from Alice was intended to have a value of $|\psi\rangle = 0.8|0\rangle + 0.6|1\rangle$, what is the value of the qubit currently in Bob’s possession (before correction)?

- a. $0.8|0\rangle + 0.6|1\rangle$
- b. $0.8|0\rangle - 0.6|1\rangle$
- c. $0.8|1\rangle + 0.6|0\rangle$
- d. $0.8|1\rangle - 0.6|0\rangle$

<table>
<thead>
<tr>
<th>Alice’s Measurement $b_1b_0$</th>
<th>00</th>
<th>01</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Bob’s qubit</td>
<td>$\alpha</td>
<td>0\rangle + \beta</td>
<td>1\rangle$</td>
<td>$\alpha</td>
</tr>
</tbody>
</table>

**Practice:**
How many entangled qubits (minimum) are needed to transmit a total of 5 message qubits with teleportation procedures?

- a. 5 entangled qubits
- b. 2 entangled qubits
- c. 10 entangled qubits
- d. 15 entangled qubits
PRACTICE: How many entangled qubits (minimum) are needed to transmit a total of 5 message qubits with teleportation procedures?

a. 5 entangled qubits
b. 2 entangled qubits
c. 10 entangled qubits
d. 15 entangled qubits

Notes about Quantum Teleportation

- Alice never knows the state of $|\psi\rangle$
- Communication is not faster than light because of the need for a classical communication channel
- Alice and Bob destroy their entangled qubits during the process of teleportation...
- more arbitrary qubit transfer requires more distributed entanglement!
- Quantum teleportation has been experimentally demonstrated many times
- Only need to transmit 2 classical bits (and start with shared entangled pair)
- Allows transmitting arbitrary $|\psi\rangle$

Applications of Teleportation

Quantum teleportation can be used in many future quantum computing tasks!
- Projected applications include:
  - Reducing computation errors
    - Noise-resistant quantum gates
  - Error correcting codes
  -Uniting quantum computers to form networks
  - Constructing ultra-secure communication channels
    - Qubits are transferred with ultimate privacy...eavesdroppers cannot read messages

The No-Cloning Theorem of QIS
Copying Information

Classical computing relies on copying information for:
- Computation
- Data storage
- Error detection and correction

Examples of copying classical data include:
- Assigning a new variable the value of an existing variable in Python
- Sending one signal down multiple wires with voltage fan-out in a classical circuit

"Cloning" a qubit

Can qubit state be cloned or duplicated?
Let’s attempt to define a “qubit copying” circuit, G:

1. **Arbitrary state to be copied:** $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$
2. **Ancilla qubit to form duplicate state:**
   - $G |0 \rangle = |00\rangle$
   - $G |1 \rangle = |11\rangle$

Resulting Quantum State after Copy Gate, $G$:

$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$

Copy Gate, $G$:

- Left side of equation: $G|\psi\rangle = G(\alpha |0\rangle + \beta |1\rangle)$
- Expand: $G|\psi\rangle = G(\alpha |0\rangle + \beta |1\rangle) = G|\psi\rangle$
- Distribute: $G|\psi\rangle = G(\alpha |0\rangle + \beta |1\rangle) = G|\psi\rangle$
- Simplify: $G|\psi\rangle = G(\alpha |0\rangle) + G(\beta |1\rangle)$
- Apply Copy Gate: $G|\psi\rangle = G(\alpha |0\rangle) + G(\beta |1\rangle)$

- Qubit state of $|\psi\rangle$ copied to ancilla qubit $|1\rangle$
Copying Arbitrary Quantum State, $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$

Right side of equation, $|\psi\rangle$ where $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$

Expand: $|\psi\rangle \otimes |\psi\rangle = (\alpha |0\rangle + \beta |1\rangle) \otimes (\alpha |0\rangle + \beta |1\rangle)$

Distribute (FOIL): $(\alpha |0\rangle + \beta |1\rangle) \otimes (\alpha |0\rangle + \beta |1\rangle) = \alpha^2 |00\rangle + \alpha \beta |01\rangle + \alpha \beta |10\rangle + \beta^2 |11\rangle$

Simplify: $\alpha^2 |00\rangle + \alpha \beta |01\rangle + \alpha \beta |10\rangle + \beta^2 |11\rangle$

Calculated value of $|\psi\rangle$

Anticipated output after state copy

Output produced by copy gate, $G$:

Anticipated output:

$\alpha^2 |00\rangle + \alpha \beta |01\rangle + \alpha \beta |10\rangle + \beta^2 |11\rangle$

There is no copy (clone) gate that can duplicate qubit state!

Takeaway: The No-Cloning Theorem

- Qubits cannot be duplicated...we call this the No-cloning Theorem
  - Cannot "see" quantum state without destroying it! Similar to measurement...
- The No-cloning Theorem has major implications on the use and storage of quantum information
- Major differences will exist for quantum versions of:
  - Algorithms
  - Error correction and detection
  - Memory

We must rethink how to solve problems as compared to classical approaches if we want to use the unique properties of QIS such as superposition!