

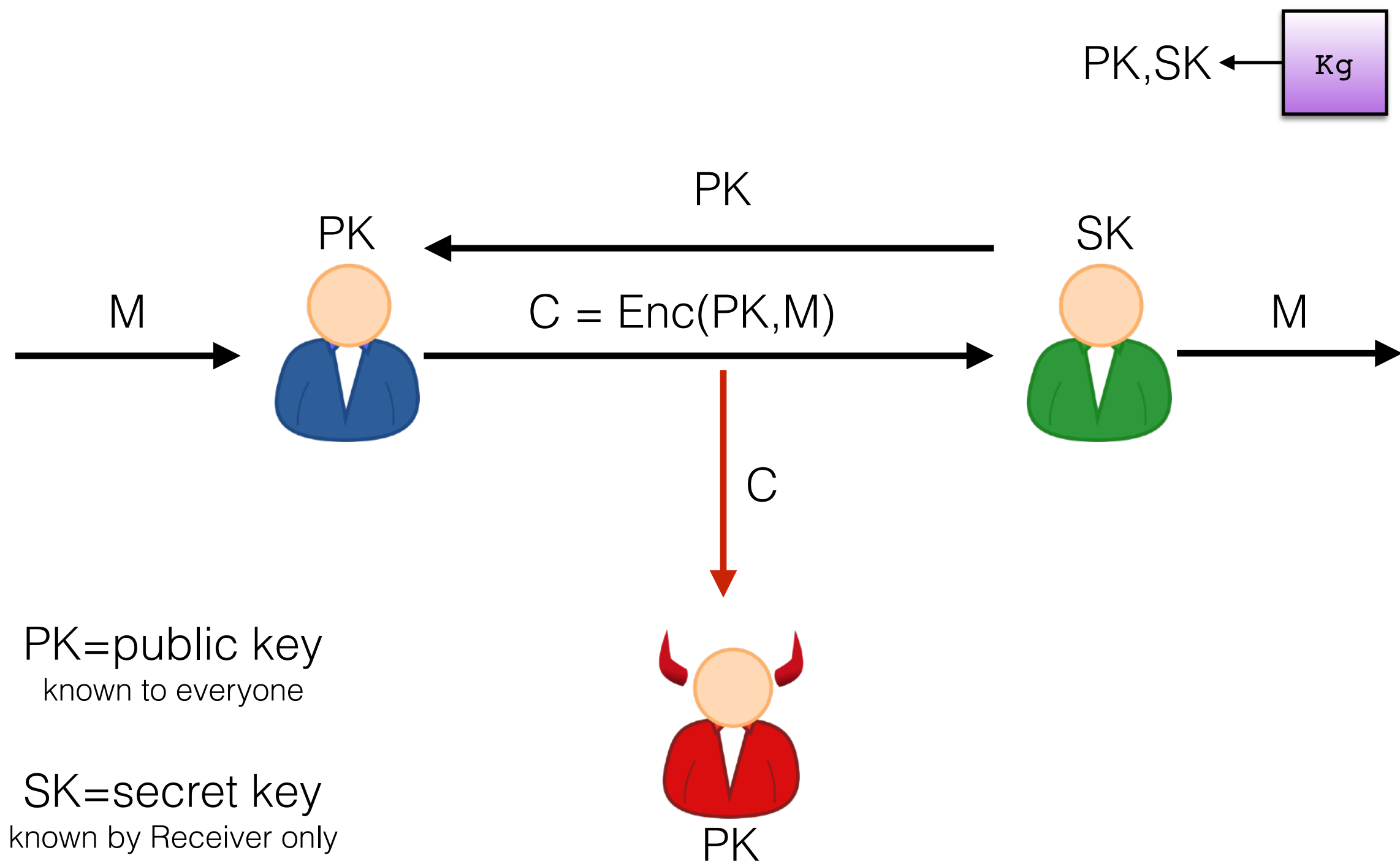
Digital Signatures, Certificates, and TLS

CMSC 23200/33250, Winter 2020, Lecture 6

David Cash and Blase Ur

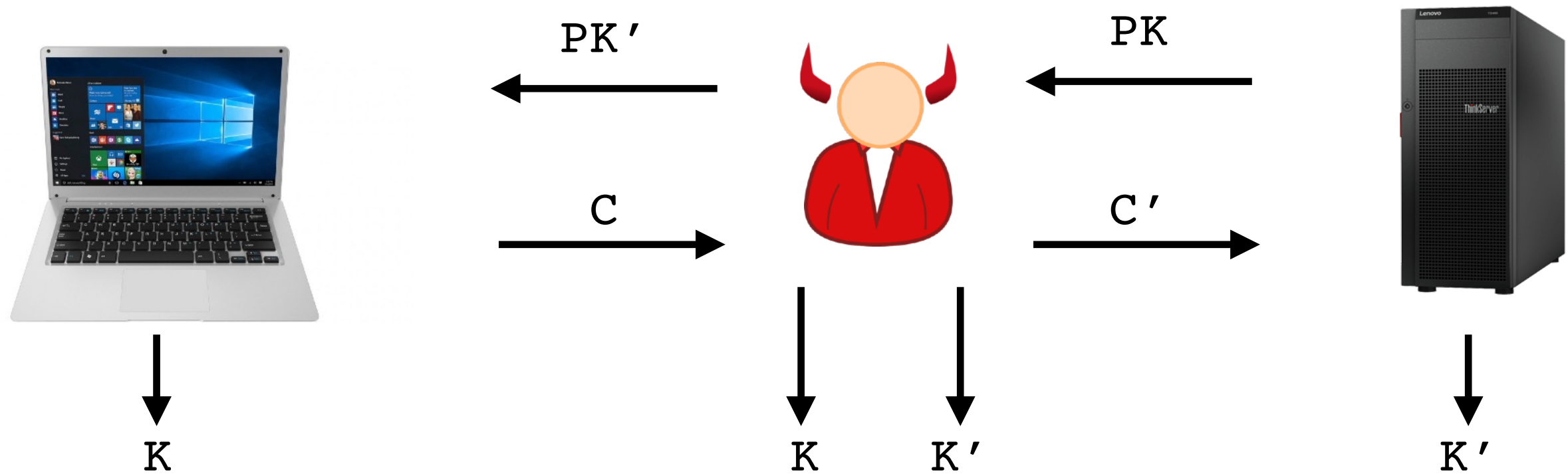
University of Chicago

Public-Key Encryption in Action



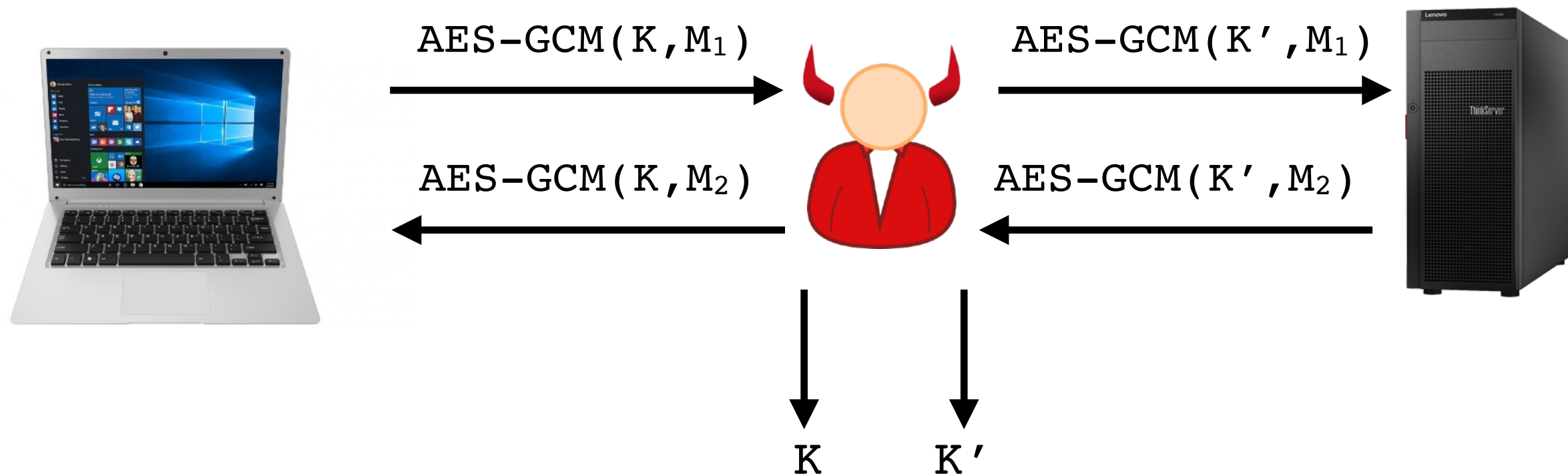
Key Exchange with a Person-in-the-Middle

Adversary may silently sit between parties and modify messages.

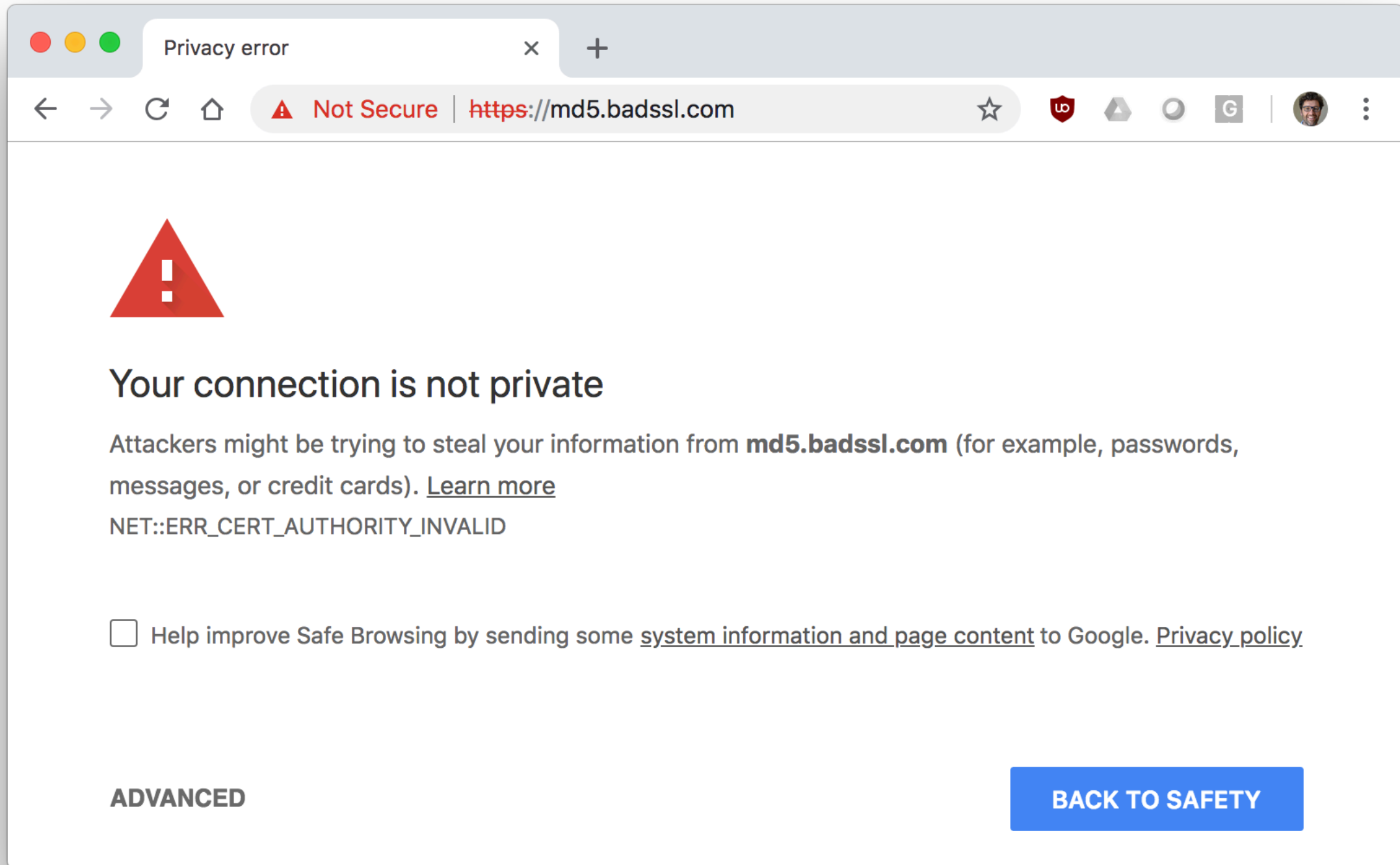


Parties agree on different keys, both known to adversary...

Key Exchange with a Person-in-the-Middle



Connection is totally transparent to adversary.
Translation is invisible to parties.



[Blog](#) >

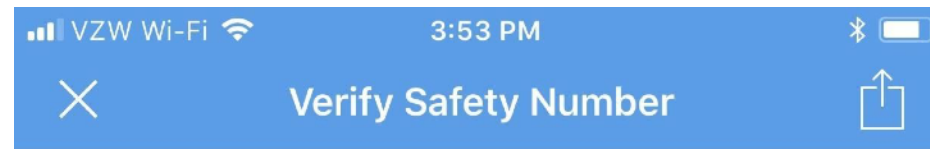
New NSA Leak Shows MITM Attacks Against Major Internet Services

The Brazilian television show "Fantastico" exposed an NSA training presentation that discusses how the agency runs man-in-the-middle attacks on the Internet. The point of the story was that the NSA engages in economic espionage against Petrobras, the Brazilian giant oil company, but I'm more interested in the tactical details.

The video on the webpage is long, and includes what I assume is a dramatization of an NSA classroom, but a few screen shots are important. The pages from the training presentation describe how the NSA's MITM attack works:

However, in some cases GCHQ and the NSA appear to have taken a more aggressive and controversial route -- on at least one occasion bypassing the need to approach Google directly by performing a man-in-the-middle attack to impersonate Google security certificates. One document published by *Fantastico*, apparently taken from an NSA presentation that also contains some GCHQ slides, describes "how the attack was done" to apparently spoof on SSL traffic. The document illustrates with a diagram how

Authenticating PK “Out of Band”



You have not marked +1 [redacted]
as verified.



Tap to Scan

27472 37554 90485 91996
35297 72831 95945 88302
31164 34110 57537 20193

If you wish to verify the security of your end-to-end encryption with +1 [redacted], compare the numbers above with the numbers on their device.

Alternatively, you can scan the code on their phone, or ask them to scan your code.

[Learn More](#)

✓ Mark as Verified

Next up: Tool for Stopping the Person-in-the-Middle

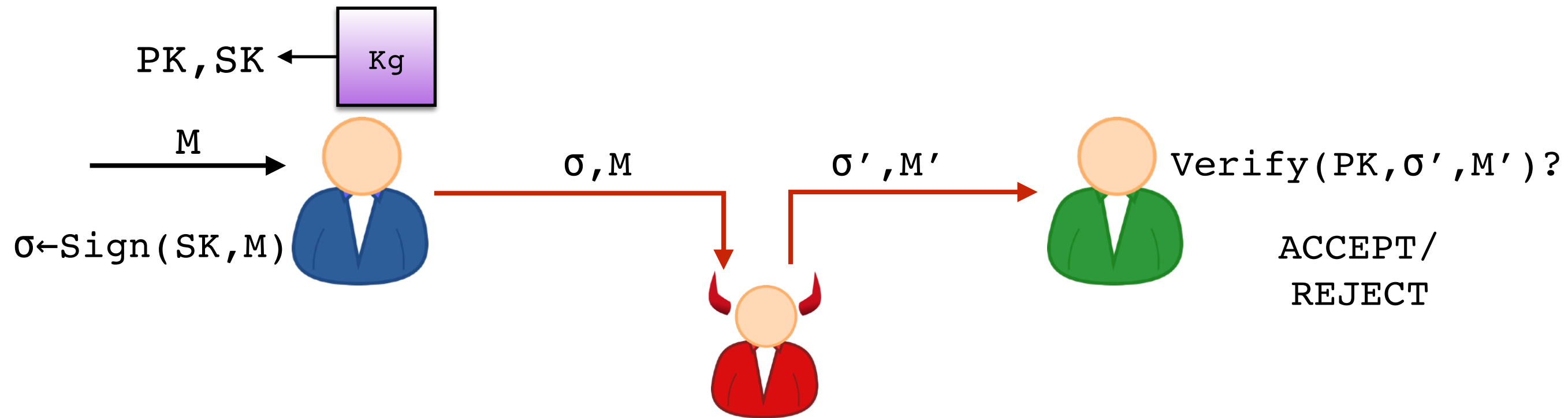
- Digital Signatures
- Public-Key Infrastructure (PKI)
- Certificates and chains of trust

Crypto Tool: Digital Signatures

Definition. A digital signature scheme consists of three algorithms **Kg**, **Sign**, and **Verify**

- Key generation algorithm **Kg**, takes no input and outputs a (random) public-verification-key/secret-signing key pair (PK, SK)
- Signing algorithm **Sign**, takes input the secret key SK and a message M , outputs “signature” $\sigma \leftarrow \text{Sign}(SK, M)$
- Verification algorithm **Verify**, takes input the public key PK , a message M , a signature σ , and outputs **ACCEPT/REJECT**
 $\text{Verify}(PK, M, \sigma) = \text{ACCEPT/REJECT}$

Digital Signature Security Goal: Unforgeability



Scheme satisfies **unforgeability** if it is unfeasible for Adversary (who knows PK) to fool Bob into accepting M' not previously sent by Alice.



Broken



“Plain” RSA with No Encoding

$$PK = (N, e) \quad SK = (N, d) \quad \text{where} \quad N = pq, \quad ed = 1 \bmod \phi(N)$$

$$\text{Sign}((N, d), M) = M^d \bmod N$$

$$\text{Verify}((N, e), M, \sigma) : \sigma^e = M \bmod N?$$

Messages & sigs
are in \mathbb{Z}_N^*

$e = 3$ is common for fast verification; Assume $e=3$ below.



Broken



“Plain” RSA Weaknesses

Assume $e=3$.

$$\text{Sign}((N, d), M) = M^d \bmod N \quad \text{Verify}((N, 3), M, \sigma) : \sigma^3 = M \bmod N?$$

To forge a signature on message M' : Find number σ' such that $(\sigma')^3 = M' \bmod N$

M=1 weakness: If $M'=1$ then it is easy to forge. Take $\sigma'=1$:

$$(\sigma')^3 = 1^3 = 1 = M' \bmod N$$



Cube-M weakness: If M' is a *perfect cube* then it is easy to forge.
Just take $\sigma' = (M')^{1/3}$; i.e. the usual cube root of M' :

Example: To forge on $M'=8$, which is a perfect cube, set $\sigma'=2$.

$$(\sigma')^3 = 2^3 = 8 = M' \bmod N$$



(Intuition: If cubing does not “wrap modulo N ”, then it is easy to un-do.)



Broken



Further “Plain” RSA Weaknesses

Assume $e=3$.

$$\text{Sign}((N, d), M) = M^d \bmod N \quad \text{Verify}((N, 3), M, \sigma) : \sigma^3 = M \bmod N?$$

To forge a signature on message M' : Find number σ' such that $(\sigma')^3 = M' \bmod N$

Malleability weakness: If σ is a valid signature for M , then it is easy to forge a signature on $8M \bmod N$.

Given (M, σ) , compute forgery (M', σ') as

$$M' = (8 * M \bmod N), \text{ and } \sigma' = (2 * \sigma \bmod N)$$

Then $\text{Verify}((N, 3), M', \sigma')$ checks:

$$(\sigma')^3 = (2 * \sigma \bmod N)^3 = (2^3 * \sigma^3 \bmod N) = (2^3 * M \bmod N) = 8M \bmod N$$

$\sigma^3 = M \bmod N$ b/c σ is valid sig. on M





Broken



Further “Plain” RSA Weaknesses

$$\text{Sign}((N, d), M) = M^d \bmod N \quad \text{Verify}((N, 3), M, \sigma) : \sigma^3 = M \bmod N?$$

To forge a signature on message M' : Find number σ' such that $(\sigma')^3 = M' \bmod N$

Backwards signing weakness: Generate *some* valid signature by picking σ' first, and then defining $M' = (\sigma'^3 \bmod N)$

Then $\text{Verify}((N, 3), M', \sigma')$ checks:

$$(\sigma')^3 = (M' \bmod N)$$





Broken



Further “Plain” RSA Weaknesses

$$\text{Sign}((N, d), M) = M^d \bmod N \quad \text{Verify}((N, 3), M, \sigma) : \sigma^3 = M \bmod N?$$

To forge a signature on message M' : Find number σ' such that $(\sigma')^3 = M' \bmod N$

Summary:

- Plain RSA Signatures allow several types of forgeries
- It was sometimes argued that these forgeries aren't important: If M is english text, then M' is unlikely to be meaningful for these attacks
- But often they are damaging anyway

RSA Signatures with Encoding

$$PK = (N, e) \quad SK = (N, d) \quad \text{where} \quad N = pq, \quad ed = 1 \bmod \phi(N)$$

$$\text{Sign}((N, d), M) = \text{encode}(M)^d \bmod N$$

Messages & sigs are in \mathbb{Z}_N^*

$$\text{Verify}((N, e), M, \sigma) : \sigma^e = \text{encode}(M) \bmod N?$$

`encode` maps bit strings to numbers in \mathbb{Z}_N^*

Encoding needs to address:

- Small M or M = perfect cube
- Malleability
- Backwards signing

Encoding must be chosen with extreme care.



Broken



RSA Signature Padding: PKCS #1 v1.5

Note: We already saw PKCS#1 v1.5 *encryption* padding. This is *signature* padding. It is different.

N: n-byte long integer.

H: Hash function.

hash_id: Magic number assigned to H

Ex: for H=SHA-256,
hash_id = 3051...0440

Sign((N,d),M):

1. digest ← hash_id || H(M) // m bytes long
2. pad ← FF || FF || ... || FF // n-m-3 'FF' bytes
3. X ← 00 || 01 || pad || 00 || digest
4. Output $\sigma = X^d \bmod N$

Verify((N,3),M, σ):

1. $X \leftarrow (\sigma^3 \bmod N)$
2. Parse $X \rightarrow aa || bb || Y || cc || digest$
3. If $aa \neq 00$ or $bb \neq 01$ or $cc \neq 00$
or $Y \neq (FF)^{n-m-3}$
or $digest \neq hash_id || H(M)$:
Output REJECT
4. Else: Output ACCEPT

Encoding needs to address:

- Perfect cubes → The high-order bits + digest means X is large and random-looking, rarely a cube.
- Malleability → Stopped by hash, ex: $H(2 * M) \neq 2 * H(M)$
- Backwards signing → Stopped by hash: given digest, hard to find M such that $H(M) = digest$.

RSA Signature Padding: PKCS #1 v1.5

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Introduces new weakness:

- Hash collision attacks: If $H(M) = H(M')$, then ...

$$\text{Sign}((N,d),M) = \text{Sign}((N,d),M')$$

- i.e., can reuse a signature for M as a signature for M'

Now: A Buggy Implementation, with an Attack

- Padding check is often implemented incorrectly
- Enables forging of signatures on *arbitrary* messages

Real-world attacks against:

- OpenSSL (2006)
- Apple OSX (2006)
- Apache (2006)
- VMWare (2006)
- All the biggest Linux distros (2006)
- Firefox/Thunderbird (2013)
- ...
- (at least 6 more in 2018 alone)



Broken



Buggy Verification in PKCS #1 v1.5 RSA Signatures

Sign((N,d),M):

1. $\text{digest} \leftarrow \text{hash_id} || H(M)$ // m bytes long
2. $\text{pad} \leftarrow \text{FF} || \text{FF} || \dots || \text{FF}$ // n-m-3 'FF' bytes
3. $X \leftarrow 00 || 01 || \text{pad} || 00 || \text{digest}$
4. Output $\sigma = X^d \bmod N$

Verify((N,3),M,σ):

1. $X \leftarrow (\sigma^3 \bmod N)$
2. Parse $X \rightarrow \text{aa} || \text{bb} || \text{Y} || \text{cc} || \text{digest}$
3. If $\text{aa} \neq 00$ or $\text{bb} \neq 01$ or $\text{cc} \neq 00$
or $\text{Y} \neq (\text{FF})^{n-m-3}$
or $\text{digest} \neq \text{hash_id} || H(M)$:
Output REJECT
4. Else: Output ACCEPT

BuggyVerify((N,3),M,σ):

1. $X \leftarrow (\sigma^3 \bmod N)$
2. Parse $X \rightarrow \text{aa} || \text{bb} || \text{rest}$
3. If $\text{aa} \neq 00$ or $\text{bb} \neq 01$:
Output REJECT
4. Parse $\text{rest} = (\text{FF})^p || 00 || \text{digest} || \dots$,
where p is *any positive number*
5. If $\text{digest} \neq \text{hash_id} || H(M)$:
Output REJECT
6. Else: Output ACCEPT

Checks if **rest** starts with any number of FF bytes followed by a 00 byte.

If so, it takes the next m bytes as digest.

Correct: X = 00 01 FF FF FF FF FF FF FF FF 00 <DIGEST>

Buggy: X = 00 01 FF 00 <DIGEST> <IGNORED BYTES>

↑
One or more FF bytes



Attacking Buggy Verification

One or more FF bytes



Buggy: $X = 00\ 01\ FF\ 00\ \langle \text{DIGEST} \rangle\ \langle \text{IGNORED} \dots\dots\dots \text{BYTES} \rangle$

To forge a signature on message M' : Find number σ' such that

$$(\sigma')^3 = 00\ 01\ FF\ 00\ H(M')\ \langle \text{JUNK} \rangle \bmod N$$

We'll use one FF byte

m bytes long

$n-m-4$ bytes free
for attacker to pick

Freedom to pick $\langle \text{JUNK} \rangle$ means we can take any σ' such that:

$$00\ 01\ FF\ 00\ H(M')\ 00\ \dots\dots\ 00 \leq (\sigma')^3 \leq 00\ 01\ FF\ 00\ H(M')\ FF\ \dots\dots\ FF$$

Sufficient to find: Any perfect cube in the given range. Then apply perfect cube attack.

Fun! (Assignment 2)

Steps in Attack

1. Pick M you want to forge a signature on.
2. Compute lower and upper bounds (numbers), using $H(M)$.
3. Find a perfect cube x within allowed range.
4. Output cube root of x as forged signature σ .

Why do so many people make this error?

- I don't *really* know for sure
- My guesses:
 - Plugging in libraries for padding removal without checks.
 - Specifically, ASN.1 parsing libraries are used to remove padding. These are overkill and programmers do not fully understand their behavior (but they also don't want to do the parsing by hand).
 - Traditional unit testing is hard to apply to crypto.
- Note: Attack (and others) defeated by using large $e=65537$
 - Example of defense-in-depth

Other RSA Padding Schemes: Full Domain Hash

N: n-byte long integer.

H: Hash fcn with m-byte output. ← Ex: SHA-256, m=32

$k = \text{ceil}((n-1)/m)$

Sign((N,d),M):

1. $X \leftarrow 00 || H(1 || M) || H(2 || M) || \dots || H(k || M)$
2. Output $\sigma = X^d \bmod N$

Verify((N,e),M, σ):

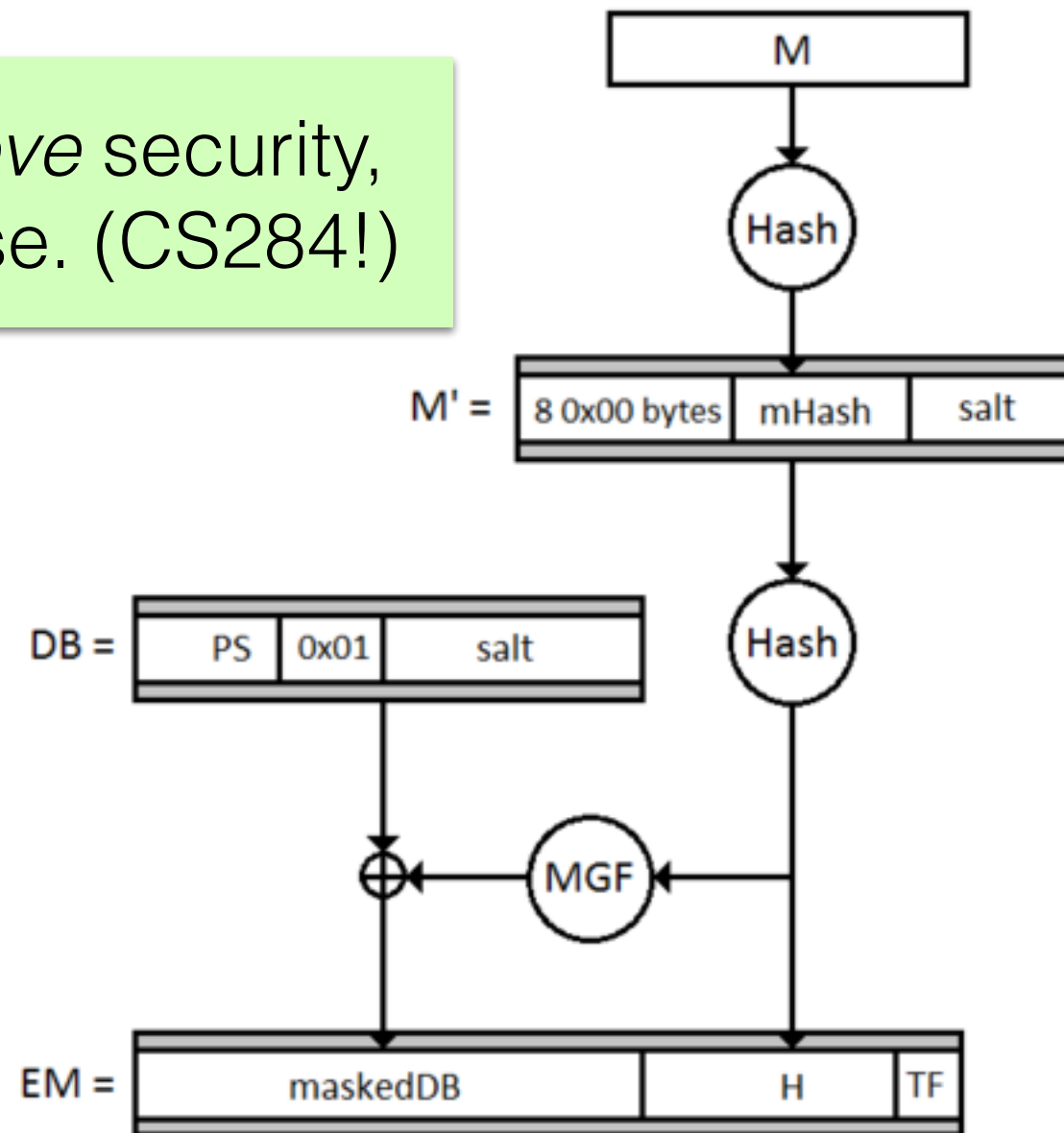
1. $X \leftarrow 00 || H(1 || M) || H(2 || M) || \dots || H(k || M)$
2. Check if $\sigma^e = X \bmod N$

Bonus: Can *prove* security,
in a strong sense.

Other RSA Padding Schemes: PSS (In TLS 1.3)

- Somewhat complicated
- *Randomized* signing

Bonus: Can *prove* security, in a strong sense. (CS284!)



RSA Signature Summary

- Plain RSA signatures are very broken
- PKCS#1 v.1.5 is widely used, in TLS, and fine if implemented correctly
- Full-Domain Hash and PSS should be preferred
- Don't roll your own RSA signatures!

Other Practical Signatures: DSA/ECDSA

- Based on ideas related to Diffie-Hellman key exchange
- Secure, but even more ripe for implementation errors

—
Hackers obtain PS3 private
cryptography key due to epic
programming fail? (update)



Sean Hollister
12.29.10

2
Shares

Sony's ECDSA code

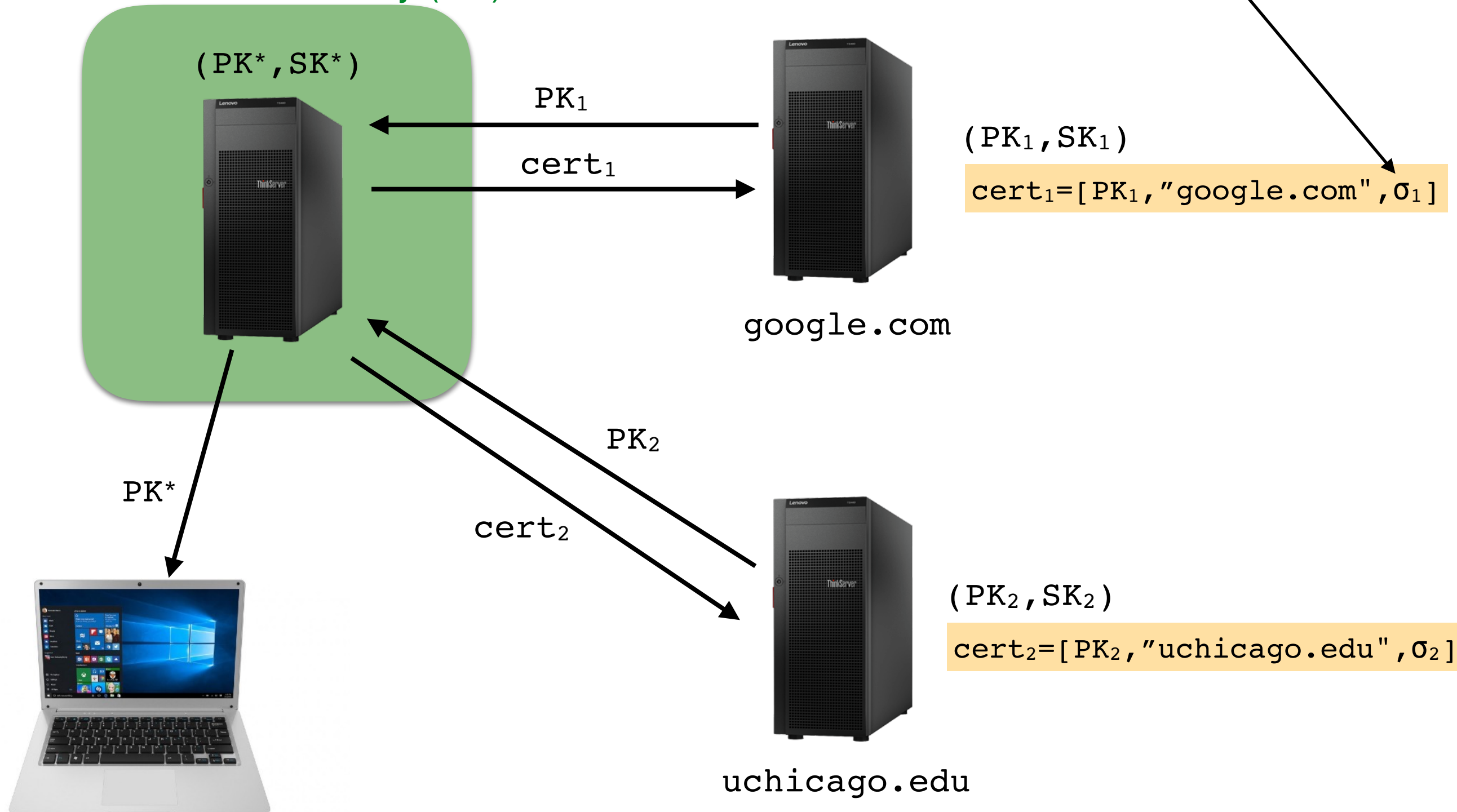
```
int getRandomNumber()  
{  
    return 4; // chosen by fair dice roll.  
              // guaranteed to be random.  
}
```

Public-Key Infrastructure (PKI)

- Main application for digital signatures are *certificates*, used in TLS and other protocols
- Used to support a “public-key infrastructure”

Certificates (Basic Idea)

Certificate Authority (CA)



- Trusted CA “issues certs”, i.e. signs public keys of other orgs.

Certificates (Basic Idea)

- Certificates in general are a tuple $(PK, metadata, \sigma)$
 - PK is public-key (may be for encryption, or for signature verification)
 - The $metadata$ domain name, company info, sometimes addresses, crypto protocols to use, expiration date, etc.
 - σ is a signature on $PK+metadata$ under CA's signing key.
- Issuing a cert involves varying levels of due diligence by CA
- If CA is negligent, then entire system is not trustworthy!

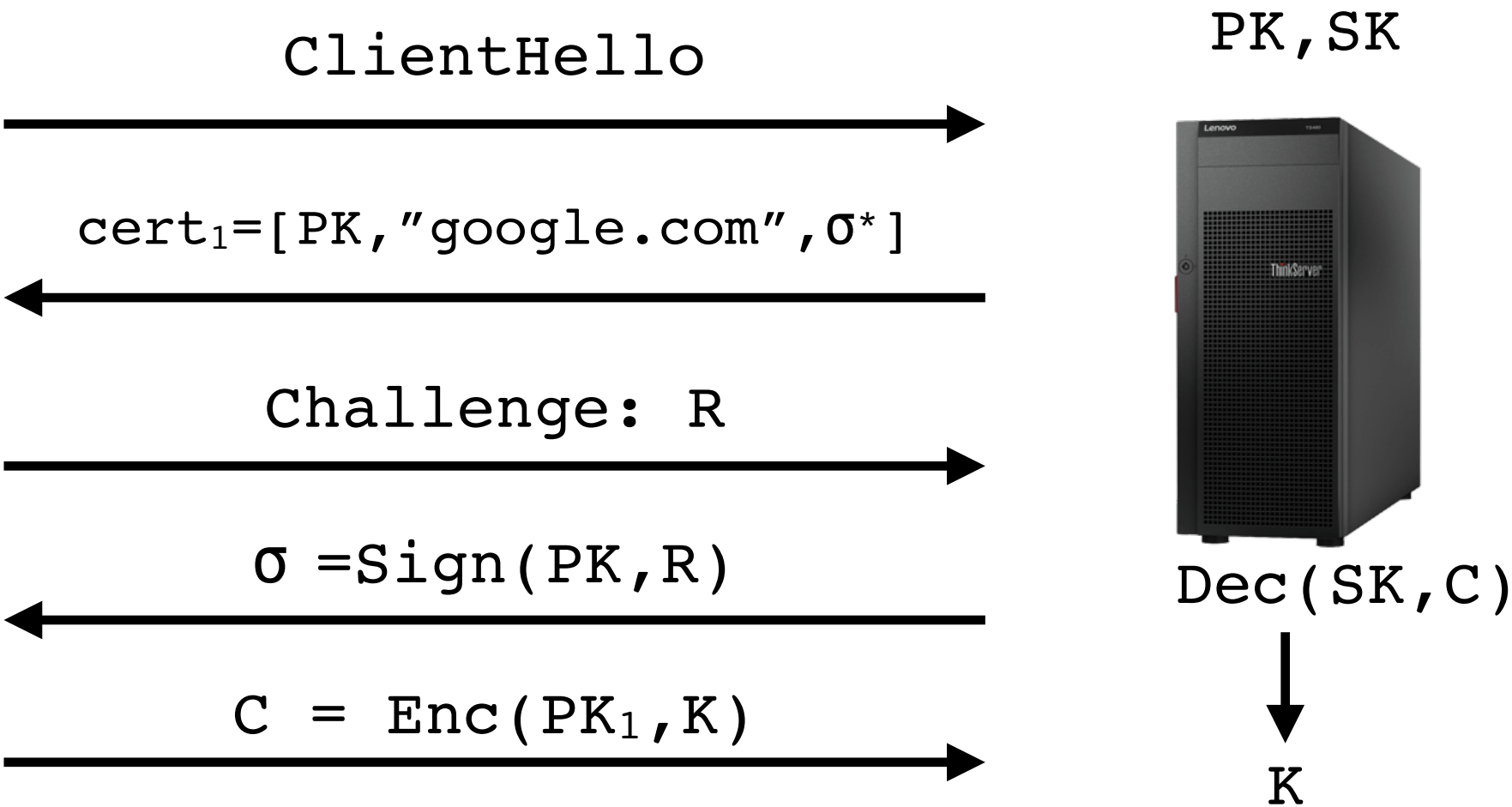
Authenticated Key Exchange with Certs

CA's verification
key PK^*

(Pick random
AES key K)



↓
 K



PK^* correct $\Rightarrow PK_1$ correct \Rightarrow Person-in-the-Middle defeated!

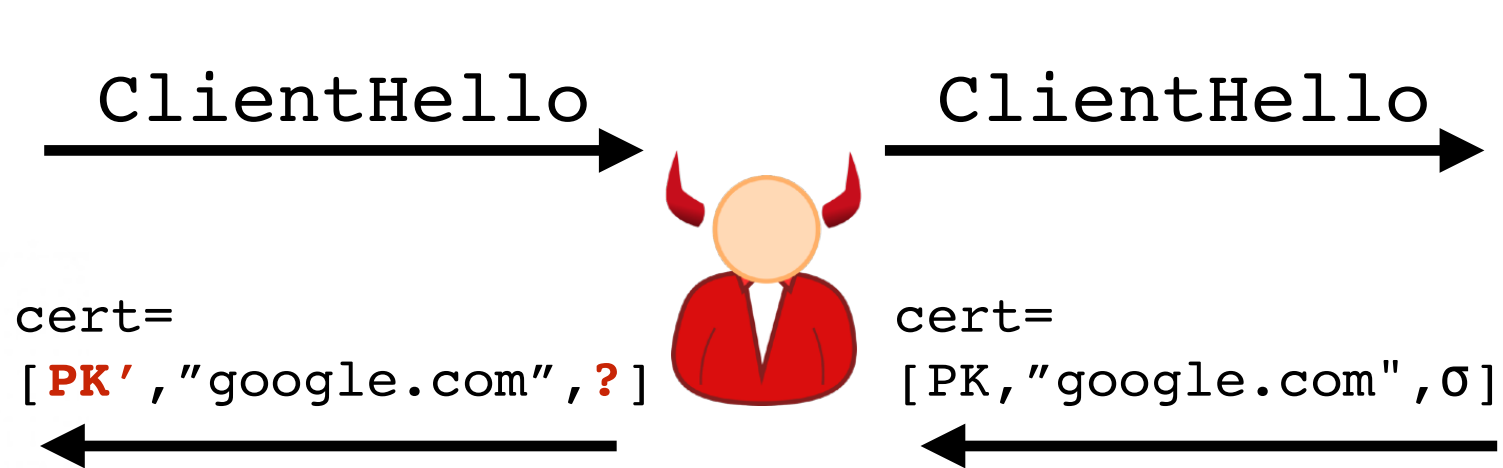
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CA's verification
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(Pick random
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↓
 K



PK, SK

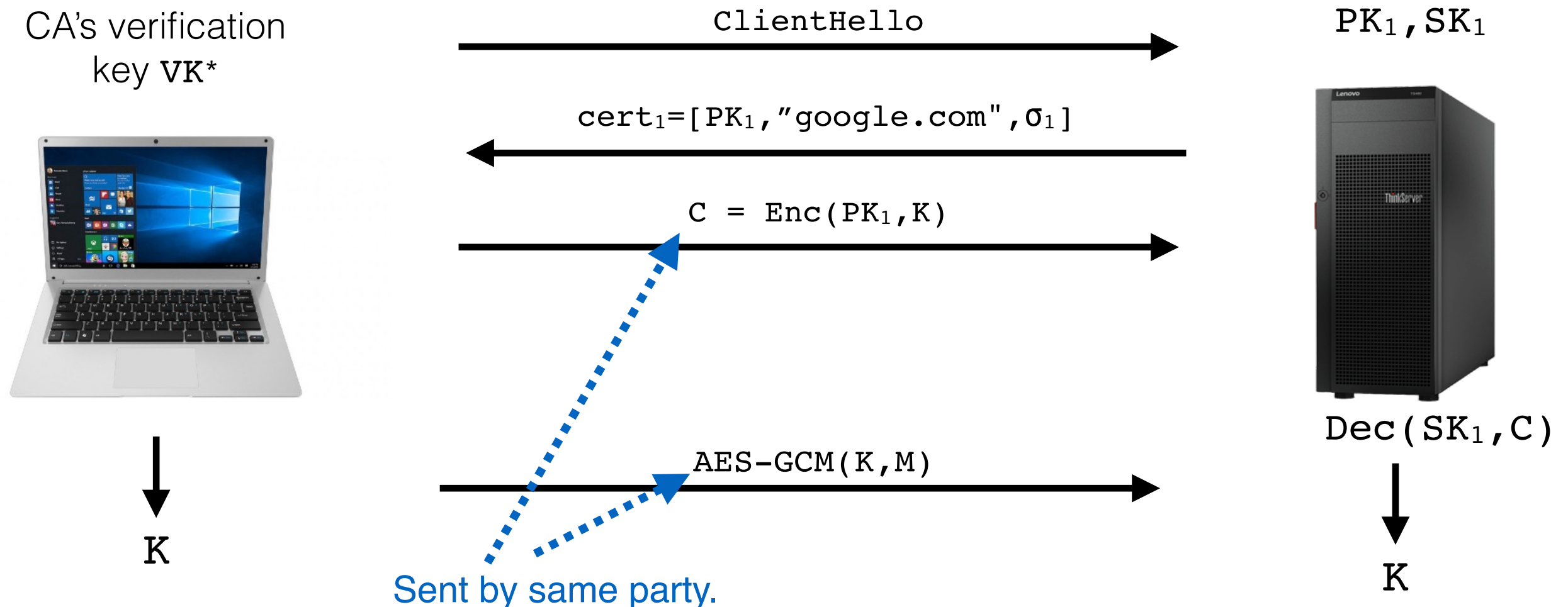


$K \leftarrow \text{Dec}(SK, C)$
↓
 K

Adversary must forge signature, or trick CA into issuing cert.

Authenticated Key Exchange Notes

- Authentication is “unilateral” or “one-sided”
 - You are convinced you’re talking to `google.com`, but `google.com` has no idea who they are talking to.
 - However `google.com` knows they are continuing to talk to whoever sent `C`
 - You convince `google.com` of your identity using a password, not TLS.



The End