CMSC 28100 Spring 2017
Homework 9

May 25, 2017

1. Show that if $\mathbf{P} = \mathbf{NP}$, then every language in $\mathbf{NP}$ is $\mathbf{NP}$-complete, except for $\emptyset$ and Σ^*.

2. Consider the following language:

$$K := \{(M, x, 1^t) \mid M \text{ is a NTM that accepts } x \text{ within } t \text{ steps}\}$$

(a) Show that $K \in \mathrm{NTIME}(n)$.

(b) Show directly (not by reduction from another known $\mathbf{NP}$-complete language) that K is $\mathbf{NP}$-complete.

3. Show that if SAT $\in \mathbf{P}$, then there is a deterministic polynomial-time Turing machine M such that for all formulas φ, if φ is satisfiable then $M(\varphi)$ outputs a satisfying assignment to φ, and otherwise M rejects. This is called solving the "search version" of SAT (searching for a witness, rather than merely determining if one exists).

4. A language L is *p-selective* if there is a polynomial-time (deterministic) Turing machine M such that 1) $M(x, y) \in \{x, y\}$ –given (x, y), M outputs either x or y– for every pair of strings (x, y), and 2) if at least one of x or y is in L, then $M(x, y)$ outputs a string in L – which is necessarily either x or y, by (1).

Show that if SAT is p-selective, then $\mathbf{P} = \mathbf{NP}$. *Hint:* The solution to the previous problem contains a relevant idea.