CS281 Spring 2011: Homework 3

Due Friday, April 29th – in class

1. (10 points)
   Informally but clearly describe a Nondeterministic Turing machine – multitape if you like – that accepts the following language [Try to take advantage of nondeterminism to avoid iteration and save time in the nondeterministic sense. That is, prefer to have your NTM branch a lot, while each branch is short.] :

   The language of all strings of the form $w_1\#w_2\#\ldots\#w_n$, for any n, such that each w_i is a string of 0's and 1's, and for some j, w_j is the integer j in binary.

2. (10 points)
   Consider the nondeterministic Turing machine

   $$M = (\{q_0, q_1, q_2, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\})$$

   Informally but clearly describe the language $L(M)$ if δ consists of the following sets of rules: $\delta(q_0, 0) = \{(q_0, 1, R), (q_1, 1, R)\}; \delta(q_1, 1) = \{(q_2, 0, L)\};$ $\delta(q_2, 1) = \{(q_0, 1, R)\}; \delta(q_1, B) = \{(q_f, B, R)\}$.

3. (10+10 = 20 points)
   State (with justification) whether the recursive languages and the RE languages are closed under the following operations. You may give informal but clear constructions to show closure.
   a) Concatenation
   b) Kleene Star[1] operation.

4. **(15 points)**
   We know by Rice's theorem that none of the following problems are decidable. However, are they recursively enumerable, or non-RE?
   a) Does $L(M)$ contain at least two strings?
   b) Is $L(M)$ infinite?
   c) Is $L(M)$ a context free language? [2]
   d) Is $L(M) = (L(M))^R$?

5. **(10 points)**
   Let L be the language consisting of pairs of TM codes plus an integer, (M_1, M_2, k), such that $L(M_1) \cap L(M_2)$ contains atleast k strings. Show that L is RE, but not recursive.

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[1] Given a language L, the Kleene star of L is the language $L^* = \bigcup_{n\in\mathbb{N}} L^n$ where $\forall n > 0$, L^n is the language consisting of concatenations of n elements of L and $L^0 = \{\Lambda\}$, Λ being the empty string.

[2] Optional, try if you know what a CFL is, it will not be graded

6. **(15 points)**
Show that the following questions are decidable:
a) The set L of codes for TM's M such that, when started with the blank tape will eventually write some nonblank symbol on its tape. *Hint*: If M has m states, consider the first m transitions that it makes.
b) The set L of codes for TM's that never make a move left on any input.
c) The set L of pairs (M, w) such that TM M, started with input w, never scans any tape cell more than once.