CS 336 Winter 2005

Type Systems

Handout 1 January 3, 2005

Basic Mathematical Concepts and Notations

sets

empty set: \emptyset singleton set: $\{a\}$ family of sets: $\{S_i|i\in I\}$ union: $A\bigcup B,\ \bigcup_{i\in I}A_i$ intersection: $A\bigcap B,\ \bigcap_{i\in I}A_i$ difference: $A\backslash B,\ A-B$

disjoint sets: $A \cap B = \emptyset$

subset: $A \subseteq B$

proper subset: $A \subset B$

powerset: $\mathcal{P}(A)$

pair, triple, n-tuple: (a,b), (a,b,c), (a_1,\ldots,a_n) Cartesian product: $A \times B$, $A \times B \times C$, $\prod_{i \in I} A_i$

relations

binary relation: $R \subseteq A \times B$, R(a,b) binary relation on a set S: $R \subseteq S \times S$ n-ary relation: $R \subseteq A_1 \times \ldots \times A_n$

domain of a relation: dom(R)

range (codomain) of a relation: range(R) composition of binary relations: $R_1 \circ R_2$ image of a set under a binary relation: R(A)

inverse image of a set under a binary relation: $R^{-1}(A)$

reflexive relation
irrefexive relation
transitive relation
symmetric relation
antisymmetric relation
reflexive closure of a relation

transitive closure of a relation: \mathbb{R}^+

reflexive, transitive closure of a relation: R^*

functions (single-valued binary relation)

domain of a function: dom(f)

range (codomain) of a function: range(f)

partial function total function

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composition of functions: f \circ g
one-to-one function (injective function, injection)
onto function (surjective function, surjection)
one-to-one, onto function (bijective function, bijection)
image (of a set or element) under a function: f(A)
inverse image (of a set or element) under a function: f^{-1}(A)
inverse of a function: f^{-1}
equivalence relations: a \equiv b
   reflexive, symmetric, transitive relation
partition of a set (induced by an equivalence relation)
equivalence class of an element
quotient set under an equivalence relation
congruence relation (equivalence relation preserving functions)
partial orders
preorder
   transitive, reflexive relation
partial order: \leq, \sqsubseteq
   antisymmetric preorder
strict partial order: <, \square
total order
chain (of elements of a partial order)
co-chain (of elements of a partial order)
\omega-chain
well-founded partial order
upper bound (of a set of elements)
lower bound (of a set of elements)
least-upper-bound (lub, supremum, join)
greatest-lower-bound (glb, infimum, meet)
minimal element
maximal element
bottom (least element): ⊥
top (greatest element): \top
limit of a chain (lub of a chain)
complete partial order (CPO)
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monotonic function (preserves ordering)

continuous function (preserves limits of chains)