CMSC 23700	**Introduction to Computer Graphics**	**Homework 1**
Winter 2010		**Due January 14**

1. Consider two unit vectors, $\mathbf{u}$ and $\mathbf{v}$. The *linear interpolation* between these vectors is defined to be

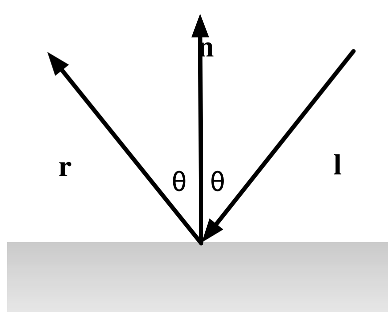
$$\mathrm{lerp}(\mathbf{u}, t, \mathbf{v}) = (1 - t)\mathbf{u} + t\mathbf{v}$$

where $0 \leq t \leq 1$. While this operation works well when the vectors represent positions, it does not work well when the vectors represent directions, since the angle between $\mathbf{u}$ and $\mathrm{lerp}(\mathbf{u}, t, \mathbf{v})$ is not a linear function of t.

Give psuedocode for a function $\mathit{slerp}(\mathbf{u}, t, \mathbf{v})$, where $0 \leq t \leq 1$, that returns a unit vector $\mathbf{w}$, such that the angle between $\mathbf{u}$ and $\mathbf{w}$ is a linear function of t.

Recall that the angle θ between unit vectors $\mathbf{u}$ and $\mathbf{v}$ is determined by $\theta = \arccos(\mathbf{u} \cdot \mathbf{v})$.

2. Consider the following picture, where $\mathbf{n}$, $\mathbf{l}$, and $\mathbf{r}$ are all unit vectors. Give an equation for $\mathbf{r}$ in terms of $\mathbf{n}$ and $\mathbf{l}$ (*i.e.*, that does not refer to θ).



3. Prove that for any three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \Re^3$,

$$\mathbf{u} \times \mathbf{v} \times \mathbf{w} = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$$

4. Affine transformations can be represented by 4×4 homogeneous matrices with the following shape:

$$\begin{bmatrix} \mathbf{M} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$$

where $\mathbf{M}$ is a 3×3 matrix and $\mathbf{t}$ is a vector. We can use $\langle \mathbf{M} \mid \mathbf{t} \rangle$ as a more compact notation for this class of matrices. The product of two homogeneous matrices is

$$\langle \mathbf{M}_1 \mid \mathbf{t}_1 \rangle \langle \mathbf{M}_2 \mid \mathbf{t}_2 \rangle = \langle \mathbf{M}_1\mathbf{M}_2 \mid \mathbf{M}_1\mathbf{t}_2 + \mathbf{t}_1 \rangle$$

and applying the transformation to a homogeneous point is

$$\langle \mathbf{M} \mid \mathbf{t} \rangle \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix} = \mathbf{M}_1\mathbf{v} + \mathbf{t}$$

If we restrict ourselves to isotropic (uniform) scaling, rotation, and translation, then these matrices are called *SRT* transforms and have the form $\langle s\mathbf{R} \mid \mathbf{t}\rangle$, where s is a scalar and $\mathbf{R}$ is a rotation matrix. Given this notation, solve the following equations:

(a) $\langle s_1\mathbf{R}_1 \mid \mathbf{t}_1\rangle\langle s_2\mathbf{R}_2 \mid \mathbf{t}_2\rangle$

(b) $\langle s\mathbf{R} \mid \mathbf{t}\rangle \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}$

(c) $\langle s\mathbf{R} \mid \mathbf{t}\rangle^{-1}$