Introduction to Complexity Theory

April 23, 2009

Homework 4 (60 Points)

- **Problem 1 (15 points)**
  We know by Rice's theorem that none of the following problems are decidable. However, are they recursively enumerable, or non-RE?
  a) Does $L(M)$ contain at least two strings?
  b) Is $L(M)$ infinite?
  c) Is $L(M)$ a context free language? [1]
  d) Is $L(M) = (L(M))^R$?

- **Problem 2 (10 points)**
  Let L be the language consisting of pairs of TM codes plus an integer, (M_1, M_2, k), such that $L(M_1) \cap L(M_2)$ contains atleast k strings. Show that L is RE, but not recursive.

- **Problem 3 (15 points)**
  Show that the following questions are decidable:
  a) The set of codes for TM's M such that, when started with the blank tape will eventually write some nonblank symbol on its tape. *Hint*: If M has m states, consider the first m transitions that it makes.
  b) The set of codes for TM's that never make a move left on any input.
  c) The set of pairs (M, w) such that TM M, started with input w, never scans any tape cell more than once.

- **Problem 4 (20 points)** Show that the following problems are not recursively enumerable:
  a) The set of pairs (M_1, M_2) such that $L(M_1) \cap L(M_2) = \emptyset$. b) The set of triples (M_1, M_2, M_3) such that $L(M_1) = L(M_2)L(M_3)$ *i.e.* the

[1] Optional, try if you know what a CFL is, it will not be graded

language of the first is the concatenation of the languages of the other two TM's.