Introduction to Complexity Theory

April 17, 2009

Homework 3 (50 points)

- Problem 1: (10 points)
  Informally but clearly describe the Nondeterministic Turing machines-multitape if you like- that accept the following languages. Try to take advantage of nondeterminism to avoid iteration and save time in the non-deterministic sense. That is, prefer to have your NTM branch a lot, while each branch is short.
  The language of all strings of the form $w_1\#w_2\#\ldots\#w_n$, for any n, such that each w_i is a string of 0's and 1's, and for some j, w_j is the integer j in binary.

- Problem 2: (10 points)
  Consider the nondeterministic Turing machine

  $$M = (\{q_0, q_1, q_2, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\})$$

  Informally but clearly describe the language $L(M)$ if δ consists of the following sets of rules: $\delta(q_0, 0) = \{(q_0, 1, R), (q_1, 1, R)\}; \delta(q_1, 1) = \{(q_2, 0, L)\};$ $\delta(q_2, 1) = \{(q_0, 1, R)\}; \delta(q_1, B) = \{(q_f, B, R)\}$.

- Problem 3: (10 points)
  A k-head Turing Machine has k heads reading cells of one tape. A move of this TM depends on the state and on the symbol scanned by each head. In one move, the TM can change state, write a new symbol on the cell scanned by each head. and can move each head left, right or keep it stationary. Since several heads may be scanning the same cell, we assume the heads are numbered 1 through k, and the symbol written by the highest numbered head scanning a given cell is the one that actually gets written there. Prove that the languages accepted by k-head Turing Machines are the same as those accepted by ordinary TM's.

- Problem 4: (10+10 = 20 points)
  State whether the recursive languages and the RE languages are closed under the following operations. You may give informal but clear constructions to show closure.

a) Concatenation
b) Kleene Star operation (i.e. if L is a RE/recursive language, is the language $L' = \{v^* —$ i.e. set of all strings of (possibly infinite) length including the null string s.t. each element in the string $\in L$). [1]

[1] Let me know if the description of the Star operation sounds unclear and I will explain it clearly in the office hour