1. [75 points] We can augment the MinML language by adding pairs (binary Cartesian products). Concretely, this amounts to adding three new expression forms to the abstract syntax, as shown here:

\[
e := \ldots \mid (e, e) \mid \text{fst}(e) \mid \text{snd}(e)
\]

The basic pair expression has the form \((e_1, e_2)\), where \(e_1\) and \(e_2\) are arbitrary expressions. Its value is a pair made up of the values of \(e_1\) and \(e_2\). The expression \(\text{fst}(e)\) projects out the first component of the pair denoted by \(e\), while \(\text{snd}(e)\) yields the second component. Thus if \(v = (2, \text{true})\), then \(\text{fst}(v) = 2\) and \(\text{snd}(v) = \text{true}\). Note that the first and second components of a pair can have different types, and also that those types can be arbitrary: a pair can have primitive values, functions, or pairs as components.

The definition of a value is also extended to include pair values:

\[
v := \ldots \mid (v, v)
\]

i.e., a pair of values is a value.

The type expressions are correspondingly extended with a product form:

\[
\tau := \ldots \mid \tau \ast \tau
\]

As with the function arrow operator, the product operator for types is written using infix notation.

(a) [10 points]. Add new typing rules for the three new expression forms (note that intuitively, a value like \((2, \text{true})\) has the product type \(\text{int} \ast \text{bool}\)).

(b) [15 points]. Add new small-step evaluation rules for the transition relation \(\rightarrow\) to cover the new expression forms. Evaluation of a pair expression should be left-to-right, as it is for the arguments of \(\text{plus}\) and \(\text{apply}\). [Hint: there will be only 6 new rules, two of which will be instructions.]

(c) [10 points]. State the new clauses in the Inversion Theorem (Theorem 9.1, p. 53) and the Canonical Forms Lemma (Lemma 10.2, p. 61) needed to deal with pairs.

(d) [20 points]. Give the new case of the proof of the Progress Theorem relating to pair expressions of the form \((e_1, e_2)\).

(e) [20 points]. Give the new cases of the Preservation Theorem relating to expressions of the form \(\text{fst}(e)\).

In parts (d) and (e), make sure the presentation of the argument is clear, correct, and complete.