CS 281 – Homework 6

This assignment is due on **Wednesday, June 4** at the beginning of class.

Definition 1. For all $k \geq 1$, $L \in \Sigma_k$ if and only if there are a polynomial $q(n)$ and a relation $R(x, y_1, \ldots, y_k)$ in P such that

$$x \in L \Leftrightarrow \exists y_1 \forall y_2 \ldots Q_k y_k R(x, y_1, \ldots, y_k)$$

where the quantifies alternate ($Q_k = \forall$ if k is even, and $Q_k = \exists$ if k is odd) and $y_1, \ldots, y_k$ range over words of length $\leq q(|x|)$. Dually, $L \in \Pi_k$ if and only if

$$x \in L \Leftrightarrow \forall y_1 \exists y_2 \ldots Q'_k y_k R'(x, y_1, \ldots, y_k)$$

With this definition, notice that, you can view a series of alternating quantifiers as a Σ or Π machine, whichever is appropriate.

- The following are equivalent for all $k \geq 1$: (10 pts)
    1. $\Sigma_k = \Sigma_{k+1}$
    2. $\Pi_k = \Pi_{k+1}$
    3. $\Sigma_k = \Pi_k$
- If for some $k \geq 1, \Sigma_k = \Pi_k$, then for all $j \geq k, \Sigma_j = \Pi_j = \Sigma_k$. Hint: first show that $\Sigma_k = \Pi_{k+1}$ and $\Pi_k = \Sigma_{k+1}$. (10 pts)
- Exercise 11.2 (20 pts)
  I was not able to get the figure online so instead use this figure below. For each of these problems (1-4) give an algorithm for membership in the complexity class you suggest.

  This algorithm will be a proof of upper bound, because it shows that the problem belongs to the proposed class. Also, by placing them in the correct set in the figure, you claim a certain lower bound under the given assumptions. However, you do not have to prove your claim. In the statement of the first two problems, the word "length" should be replaced with "weight".