1. Note: No late homeworks will be accepted after Dec. 3rd.

2. A Hamiltonian path in a graph $G$ is an ordering of the vertices \{${v_1, v_2, \ldots, v_n}$\} such that \{{$v_j, v_{j+1}$}\} is an edge for all $1 \leq j \leq n - 1$. ($v_1$ and $v_n$ do not need to be adjacent this would give a Hamiltonian cycle.)

A tournament is a directed graph such that for each pair of vertices $u, v$, exactly one of $(u, v)$ and $(v, u)$ is an edge of the graph.

Prove that every tournament has a Hamiltonian path.

3. Define the compliment of a graph $G = (V, E)$ to be the graph $\bar{G} = (V, \bar{E})$. (Namely it has the complimentary set edges.) Prove that if $G$ is a simple graph with at least 11 vertices, then either $G$ or $\bar{G}$ is nonplanar.

4. Prove that if $T$ is a stochastic matrix then $T^k$ is stochastic for all positive integers $k$.

5. For any graph, we can form a Markov chain by letting the transition from vertex $v$ to any of its neighbors be $1/\text{deg}(v)$. Prove that such a Markov chain is periodic if and only if the graph is bipartite.

6. Say you have $n$ homework problems and you have already solved $k$ of them. Everyday you pick one problem equally at random to look at. If you haven’t already, you solve the problem. Let $X_i$ represent the event that $i$ problems are unsolved.

(a) Prove that this process is a Markov chain with states $X_i$.
(b) What is the transition matrix for this chain?
(c) What states are recurrent, transient, periodic?
7. Assume the Markov chain of Figure 1 is in state 0 just before the first step.
   (a) What states are recurrent, transient, periodic?
   (b) What is the probability that after 6 steps you are in state 3?
   (c) What is the probability that after 6 steps you are in state 5?
   (d) What is the expected number of steps before the process leaves state 0?
   (e) What is the probability that we never get to state 6?
   (f) For each ergodic recurrence class, find the stationary distribution.