1. (12 points) Let $a_n = \ln(n!)$ and $b_n = n((\ln n) - 1)$. Let $d_n = a_n - b_n$. Asymptotically evaluate $d_n$. Find constants $r, s, t$ such that $d_n \sim r n^s (\ln n)^t$. Determine the values $r, s, t$. Prove your answer.

2. (9 points) A complete binary tree has $n$ nodes. Determine the exact height of the tree. Prove your answer.

3. (15 points) Describe in pseudocode the INCREASE-KEY($x$, newkey) operation in a heap. ($x$ is the name of a node; newkey is the value with which we replace key($x$). We assume newkey > key($x$).)
4. (24 points) $n$ coins of various values are arranged on a line. Two players take turns to remove a coin from an end of the line until all coins have been removed. Each player in their turn removes exactly one coin and the coin must come either from the right end or the left end of the line so the remaining coins remain contiguous. Each player wants to maximize the total value of the coins collected. Find the optimal move for the first player in $O(n^2)$ steps. (Each player plays optimally.) Hint: dynamic programming.

Example: if the coin values are 10, 25, 10, 1 cents then Player 1 should pick the penny because picking the dime would permit Player 2 to pick the quarter. Now out of 10, 25, 10, no matter what Player 2 picks, the quarter will go to Player 1. So if each player plays optimally, Player 1 will collect 26 cents, while Player 2 gets 20.