Homework 2 - Due Wednesday October 10th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

1. Give a bijection between the collection of odd subsets (subsets of odd size) of \([n]\) and the collection of even subsets (subsets of even size) of \([n]\).

2. How many functions \(f : [n] \to [n]\) are there that are monotone, that is for \(i < j\) we have \(f(i) \leq f(j)\)?

3. How many positive integers strictly less than 2102 are divisible by 2, 3, or 5?

4. Let \(n \geq 7\). count the number of 0/1 strings of length \(n\) that contain at least \(n - 3\) consecutive 1s?

5. Let \(a_n\) be the sequence for the Fibonacci numbers: \(a_0 = 0, a_1 = 1,\) and \(a_n = a_{n-1} + a_{n-2}\) for \(n \geq 2\). Find a simple closed expression for the ordinary generating function \(F(x) = \sum_{n \geq 0} a_n x^n\).

6. Prove that \(\frac{x^2 + x}{(1-x)^4}\) is the ordinary generating function for the sequence \(a_n = 1^2 + 2^2 + \ldots + n^2\). Use the generating function to find a closed form for the sum \(1^2 + 2^2 + \ldots + n^2\).