Algorithms – CS-37000 Homework 5 – January 19, 2006

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ADVICE. Take advantage of the TA's problem sessions.

HOMEWORK. Please **print your name on each sheet.** Put every solution on a separate sheet. Please try to make your solutions easily readable.

This homework is due on **Tuesday, January 24** at the **beginning of the class**.

5.1 (6 points) Draw a weighted DAG (directed acyclic graph: directed graph with no directed cycles) with as few vertices and edges as possible, with one edge of negative weight, all other edges having positive weights, such that Dijkstra's algorithm fails on this input. For each of the variables "status," "cost" (c), "parent" (p), make a table (so you need to make 3 tables) showing the progress of that variable during the execution of Dijkstra's algorithm on your DAG. The rows of each table should be numbered $0, 1, 2, \ldots$ indicating the rounds (executions of the while loop); the columns should correspond to the vertices. The entry in row i, column x should be the value of the variable in question on node x after completing the loop for the i-th time. Row 0 should indicate the value of the variable after initialization, just before entering the while loop for the first time. So for example row 0 in the cost table has 0 at the source node and ∞ everywhere else.

5.2 (8 points) Draw a weighted *undirected* graph with positive weights with a source vertex s and a target vertex t such that during the course of executing Dijkstra's algorithm, the cost $c(t)$ is reduced at least 5 times (from its initial value $c(t) = \infty$). Your graph should have as few edges as possible. Indicate in 3 tables (as in the previous exercise) the progress of the variables "status," c, p as in the previous problem (one table for each variable).