

Introduction to Complexity Theory – CS-28100
Homework 4 – April 21, 2006
Instructor: Ketan Mulmuley Ry-165B

HOMEWORK. Please **print your name on each sheet**. Please try to make your solutions readable.

This homework is due on **Friday, April 28** at the **beginning of the class**.

- 4.1 Suppose we have a two-tape Turing machine, where one tape is an input tape, which cannot be written to, and the other tape is of a fixed finite length. Also, suppose that we can move right but cannot move left on the input tape.

Prove that any language recognizable by such a Turing machine is a regular language.
- 4.2 Prove that the statement of the previous exercise is false if we allow the head on the input tape to move in either direction. That is, define a language that can be recognized by such a machine, and prove that it is not regular. (Hint: use the Pumping Lemma.)
- 4.3 Prove that if a language L can be recognized by a nondeterministic finite automaton, it can be recognized by a deterministic finite automaton.
- 4.4 Prove that the language $\{ww \mid w \in \{0,1\}^*\}$ cannot be generated by a context-free grammar. (Hint: use the Pumping Lemma for context-free grammars.)
- 4.5 Show that PCP is undecidable even if we limit the alphabet to $\Sigma = \{0,1\}$ by reducing PCP to this special case of PCP. (Exercise 9.4.2 from Hopcroft, Mulwani, Ullman)
- 4.6 Suppose we limited PCP to a one-symbol alphabet. Would this restricted case of PCP still be undecidable? (Exercise 9.4.3 from Hopcroft, Mulwani, Ullman)