

Introduction to Complexity Theory – CS-28100  
Homework 3 – April 14, 2006  
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HOMEWORK. Please **print your name on each sheet**. Please try to make your solutions readable.

This homework is due on **Wednesday, April 19** at the **beginning of the class**.

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3.1 Given a function  $f$ , let  $L_f = \{(x, f(x)) \mid x \in \Sigma^*, f \text{ defined on } x\}$ . Then  $f$  is a total computable function iff  $L_f$  is recursive. Also  $f$  is a partial computable function iff  $L_f$  is recursively enumerable.

3.2 Prove that an infinite set is decidable iff it can be enumerated in increasing order by a one-to-one computable function. (Homework 3.4 from the book of Homer and Selman)

3.3 Let  $L_1, L_2, \dots, L_k$  be a collection of languages over  $\Sigma$  such that:

1. For all  $i \neq j$ ,  $L_i \cap L_j = \emptyset$ ; *i.e.* no string is in two languages.
2.  $L_1 \cup L_2 \cup \dots \cup L_k = \Sigma^*$ ; *i.e.* every string is in one of the languages.
3. Each of the languages  $L_i$ , for  $i = 1, 2, \dots, k$  is r.e.

Prove that each of the languages is therefore recursive. (Exercise 9.2.4 from the book of Hopcroft, Motwani & Ullman)

3.4 Tell (with proof) whether the recursive languages and/or the r.e. languages are closed under the following operations. You may give informal, but clear, constructions to show closure.

- a. Union
- b. Intersection
- c. Concatenation
- d. Kleene closure (star).
- e. Homomorphism
- f. Inverse homomorphism

(Exercise 9.2.6 from the book of Hopcroft, Motwani & Ullman)

3.5 Prove that every infinite c.e. (r.e.) set contains an infinite decidable (recursive, computable) subset. (Homework 3.6 from the book of Homer and Selman)