CMSC-37110 Discrete Mathematics: Second Quiz
11-28-2006

Name (print): ____________________________________

Show all your work. Do not use book or notes. Do not use separate sheets, write your answers in the space provided after each question. If you are not sure you understand a problem properly, **ask the instructor.** The BONUS PROBLEMS are undervalued, do not solve them until you solved the regular problems. This quiz contributes 4% to your course grade.

1. (6 points) Give a closed-form expression for the number A_n of those n-digit integers which have only odd digits $(1, 3, 5, 7, 9)$ and all the five odd digits actually occur. Name the method used.

2. (6 points) Let p be a prime. Suppose p^k divides $n!$ (n-factorial). Prove: $k < \frac{n}{p-1}$.

3. (5B points: bonus problem) Prove: for almost all graphs G, $\chi(G) > \omega(G)^{100}$. ($\chi(G)$ denotes the chromatic number and $\omega(G)$ is the clique number, i. e., the size of the largest clique (complete subgraph) in G.)

4. (6 points) Prove: if a finite Markov Chain has two different stationary distributions then it has infinitely many.

(over)

5. (8 points) The sequence $t_n = 2^n + 3^n$ satisfies the recurrence $t_n = at_{n-1} + bt_{n-2}$ where a, b are constants (they don't depend on n). Determine a and b.

6. (4 points) Draw a strongly connected aperiodic digraph with as few edges as possible. Loops are not permitted. ("Aperiodic" means the g.c.d. of the lengths of all cycles is 1.)

7. (8 points) Let v be a vertex of a strongly connected aperiodic digraph G. Let d be the g.c.d. of the lengths of all closed walks starting (and ending) at v. Prove: $d = 1$.

8. (5B points: bonus problem) Prove: for every $k \geq 1$, every sufficiently large tournament contains a subtournament on k vertices which is a DAG. (A tournament is a directed complete graph: every pair of vertices is directed in exactly one direction.)