CMSC-37110 Discrete Mathematics: First Quiz 10-24-2006

Name (print): ______________________________

Show all your work. Do not use book or notes. Do not use separate sheets, write your answers in the space provided after each question. You may use a pocket calculator for basic arithmetic only (no binomial coefficients, etc.). If you are not sure you understand a problem properly, **ask the instructor.** The BONUS PROBLEMS are undervalued, do not solve them until you solved the regular problems.

This quiz contributes 4% to your course grade.

1. (6 points) What is the expected number of Kings in a poker-hand? Prove your answer. Make sure you give a clear definition of each of the random variables you introduce. 4 out of the 6 points go for the definition.

2. (5 points) For what values of x is the following statement true:

   $(\forall y)($ if $x \mid 12y$ then $x \mid y)$.

   Prove your answer. You must prove (a) that the good values of x are indeed good; and (b) the bad values are indeed bad.

3. (3 points) Give an example of two sequences of positive numbers, a_n and b_n, such that $a_n \sim b_n$ but $2^{a_n} \not\sim 2^{b_n}$.

4. (2+3 points) Write the following statements as properly and fully quantified formulas with no English words except IF, THEN, AND, PRIME: **(a)** "For all sufficiently large x, the quantity $\pi(x)$ is greater than

$x/(2\ln x)$." **(b)** "All primes of the form $4k+1$ can be written as the sum of two squares." (Example: $29 = 4 \cdot 7 + 1 = 5^2 + 2^2$.)

5. (2 points) Evaluate the following sum as a closed-form expression: $\displaystyle\sum_{k=0}^{n} 3^{-k/2}$.

6. (5 points) Count the n-digit integers with *non-decreasing* digits. (E.g. if $n = 4$ then 2357 and 2337 count, but 3454 does not count.) Your answer should be a binomial coefficient. Do not evaluate.

7. (5 points) If two events, A and B, are positively correlated, what can we say about their complements, $\overline{A}$ and $\overline{B}$? Prove your answer.

8. (BONUS PROBLEM, 4 points) Alice and Bob each flip n coins. What is the probability that they get an equal number of heads? Give a simple closed-form expression.