Name (print): __________________________

Show all your work. Do not use book or notes. Do not use separate sheets, write your answers in the space provided after each question. You may use a pocket calculator for basic arithmetic only (no binomial coefficients, etc.). If you are not sure you understand a problem properly, ask the instructor. The BONUS PROBLEMS are undervalued, do not solve them until you solved the regular problems.

This quiz contributes 4% to your course grade.

1. (6 points) What is the expected number of Kings in a poker-hand? Prove your answer. Make sure you give a clear definition of each of the random variables you introduce. 4 out of the 6 points go for the definition.

2. (5 points) For what values of $x$ is the following statement true:

$$(\forall y)(\text{if } x \mid 12y \text{ then } x \mid y).$$

Prove your answer. You must prove (a) that the good values of $x$ are indeed good; and (b) the bad values are indeed bad.

3. (3 points) Give an example of two sequences of positive numbers, $a_n$ and $b_n$, such that $a_n \sim b_n$ but $2^{a_n} \not\sim 2^{b_n}$.

4. (2+3 points) Write the following statements as properly and fully quantified formulas with no English words except IF, THEN, AND, PRIME:

(a) “For all sufficiently large $x$, the quantity $\pi(x)$ is greater than
5. (2 points) Evaluate the following sum as a closed-form expression:
\[ \sum_{k=0}^{n} 3^{-k/2}. \]

6. (5 points) Count the \( n \)-digit integers with non-decreasing digits. (E.g. if \( n = 4 \) then 2357 and 2337 count, but 3454 does not count.) Your answer should be a binomial coefficient. Do not evaluate.

7. (5 points) If two events, \( A \) and \( B \), are positively correlated, what can we say about their complements, \( \overline{A} \) and \( \overline{B} \)? Prove your answer.

8. (BONUS PROBLEM, 4 points) Alice and Bob each flip \( n \) coins. What is the probability that they get an equal number of heads? Give a simple closed-form expression.