Solution:
(a) \(((\neg P) \land Q) \rightarrow (P \lor R)\)
(b) \(((P \lor ((\neg Q) \land R)) \rightarrow (P \lor R)) \rightarrow (\neg Q)\)
(c) \((A \rightarrow (B \lor (((\neg C) \land D) \land E))) \rightarrow F\)

2. [5] Use truth tables to verify the equivalence \(A \lor (A \land B) \equiv A\).
Solution:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>A \land B</td>
<td>A \lor (A \land B)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>-----------</td>
<td>--------------------</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

3. [10] Exercise 6.2.8(b) (p. 367)
Solution:
Let \(W_1 = (A \lor B) \land (A \rightarrow C) \land (B \rightarrow D) \rightarrow (C \lor D)\).

\[
W_1(A/\text{true}) = (\text{true} \lor B) \land (\text{true} \rightarrow C) \land (B \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv \text{true} \land C \land (B \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv C \land (B \rightarrow D) \rightarrow (C \lor D)
\]

Let \(W_2 = C \land (B \rightarrow D) \rightarrow (C \lor D)\).

\[
W_2(B/\text{true}) = C \land (\text{true} \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv C \land D \rightarrow (C \lor D)
\]

Let \(W_3 = C \land D \rightarrow (C \lor D)\).

\[
W_3(C/\text{true}) = \text{true} \land D \rightarrow (\text{true} \lor D)
\]
\[
\equiv D \rightarrow \text{true}
\]
\[
\equiv \text{true}
\]

\[
W_3(C/\text{false}) = \text{false} \land D \rightarrow (\text{false} \lor D)
\]
\[
\equiv \text{false} \rightarrow D
\]
\[
\equiv \text{true}
\]
Therefore, $W_3$ is a tautology.

\[
W_2(B/\text{false}) = C \land (\text{false} \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv C \land \text{false} \rightarrow (C \lor D)
\]
\[
\equiv C \rightarrow (C \lor D)
\]

Let $W_4 = C \rightarrow (C \lor D)$.

\[
W_4(C/\text{true}) = \text{true} \rightarrow (\text{true} \lor D)
\]
\[
\equiv \text{true} \rightarrow \text{true}
\]
\[
\equiv \text{true}
\]

\[
W_4(C/\text{false}) = \text{false} \rightarrow (\text{false} \lor D)
\]
\[
\equiv \text{false} \rightarrow D
\]
\[
\equiv \text{true}
\]

Therefore, $W_4$ is a tautology. Because $W_3$ and $W_4$ are tautologies, $W_2$ is a tautology.

\[
W_1(A/\text{false}) = (\text{false} \lor B) \land (\text{false} \rightarrow C) \land (B \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv B \land \text{true} \land (B \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv B \land (B \rightarrow D) \rightarrow (C \lor D)
\]

Let $W_5 = B \land (B \rightarrow D) \rightarrow (C \lor D)$.

\[
W_5(B/\text{true}) = \text{true} \land (\text{true} \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv \text{true} \land D \rightarrow (C \lor D)
\]
\[
\equiv D \rightarrow (C \lor D)
\]

Because we just proved that $W_4 = C \rightarrow (C \lor D)$ is a tautology, $D \rightarrow (C \lor D)$ is a tautology for the same reasons.

\[
W_5(B/\text{false}) = \text{false} \land (\text{true} \rightarrow D) \rightarrow (C \lor D)
\]
\[
\equiv \text{false} \rightarrow (C \lor D)
\]
\[
\equiv \text{true}
\]

Therefore, $W_5$ is a tautology. Because $W_2$ and $W_5$ are tautologies, $W_1$ is a tautology.

4. [10] Exercise 6.2.9(d) (p. 368)
Solution:

\[
A \lor B \rightarrow C \equiv (\neg(A \lor B)) \lor C
\]
\[
\equiv ((\neg A) \land (\neg B)) \lor C
\]
\[
\equiv ((\neg A) \lor C) \land ((\neg B) \lor C)
\]
\[
\equiv (A \rightarrow C) \land (B \rightarrow C)
\]
5. [10] Exercise 6.2.10(d) (p. 368)
Solution:
\[ A \rightarrow (B \rightarrow A) \equiv (\neg A) \lor (B \rightarrow A) \]
\[ \equiv (\neg A) \lor ((\neg B) \lor A) \]
\[ \equiv ((\neg A) \lor A) \lor (\neg B) \]
\[ \equiv \text{true} \lor (\neg B) \]
\[ \equiv \text{true} \]

6. [10] Exercise 6.2.11(f) (p. 368)
Solution:
\[ (A \lor B) \land (C \rightarrow D) \equiv (A \lor B) \land (\neg C \lor D) \]
\[ \equiv ((A \lor B) \land \neg C) \lor ((A \lor B) \land D) \]
\[ \equiv ((A \land \neg C) \lor (B \land \neg C)) \lor ((A \land D) \lor (B \land D)) \]
\[ \equiv (A \land \neg C) \lor (B \land \neg C) \lor (A \land D) \lor (B \land D) \]

7. [10] Exercise 6.2.12(f) (p. 368)
Solution:
\[ (A \land B) \lor E \lor F \equiv (A \land B) \lor (E \lor F) \]
\[ \equiv (A \lor (E \lor F)) \land (B \lor (E \lor F)) \]
\[ \equiv (A \lor E \lor F) \land (B \lor E \lor F) \]

8. [40] Write (and test and debug) a program in your favorite language that will evaluate the truth of a WFF given a truth assignment (or interpretation) for its propositional variables. You will need to do the following:
(a) Define a data structure representing WFFs, including propositional variables. You might, for instance, represent propositional variables as strings, or possibly as numbers (integers, say).
(b) Define a data structure or other representation for truth assignments. You could, for instance, use an association list data structure consisting of a list or sequence of ordered pairs, where each ordered pair would consist of a propositional variable and its assigned truth value. E.g. the assignment
\[ \{P \mapsto T, Q \mapsto F, R \mapsto T\} \]
would be represented by the association list (using ML list syntax):
\[ [(P, T), (Q, F), (R, T)] \]
(c) Define an evaluator that takes as parameters a WFF and a truth assignment, and returns true or false as appropriate.