Algorithms – CS-27200/37000 Homework – January 12, 2005

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IMPORTANT. If you have not done so yet, please send e-mail to the instructor with your name, major, year, type of credit sought (letter grade, P/F, etc.), list of proof-oriented math courses previously taken; include whether or not you took CMSC-27100 (Discrete Math). In the subject write 27200 info or 37000 info, as approrpiate.

HOMEWORK. Please **print your name on each sheet.** Print "U" next to your name if you seek 27200 credit and "G" if you seek 37000 credit. Undergraduates receive the stated number of points as *bonus points* for "G only" problems. – Please try to make your solutions readable. Unless expressly stated otherwise, all solutions are due at the **beginning of the next class.**

Homework is collected in three separate piles (U, G, "G only"). Please put your solutions to "G only" problems on that pile, and your solutions to other problems on the "U" or "G" pile according to the credit you seek.

3.1 (U, G) (5 points) Prove: it takes fewer than $3n/2$ comparisons to find both the maximum and the minimum of n keys from a linearly ordered universe.

3.2 (G only) (16 points. Due Wednesday, February 9.) *This is a difficult problem, start working on it right away.*

We have n coins; at least one of them is bad. All the good coins weigh the same, and all the bad coins weigh the same. The bad coins are lighter than the good coins. Find the *number* of bad coins, making $O(\log^2 n)$ comparisons on a balance. ($\log^2 n := (\log n)^2$.)

Hint. Solve the following auxiliary problem: divide the coins as evenly as possible, using $O(\log n)$ comparisons on the balance. (I.e., if n is even and b, the number of bad coins, is also even, then split the coins evenly, i. e., each tray should have $n/2$ coins, including $b/2$ bad ones. If n is even but b is odd, divide the coins into two groups of $n/2$ coins each so that the number of bad coins in one group is $(b-1)/2$ and in the other, $(b+1)/2$. If n is odd, remove a coin and split the rest as above.) (8 points for the auxiliary problem and another 8 points for showing how to use the auxiliary problem to solve the main problem.)

OPEN QUESTIONS (as far as I know). Do $o(\log^2 n)$ comparisons suffice for the main problem? Do $O(\log n)$ comparisons suffice?