CMSC-37110 Discrete Mathematics: Third Quiz
11-17-2005

Name (print): ________________________________

Show all your work. Do not use book or notes. Do not use separate sheets, write your answers in the space provided after each question. You may use a pocket calculator for basic arithmetic only (no binomial coefficients, etc.). If you are not sure you understand a problem properly, **ask the instructor.** The BONUS PROBLEMS are undervalued, do not solve them until you solved the regular problems.

This quiz contributes 6% to your course grade.

1. (3+3 points) (a) Draw a topological K_4 with 7 vertices. (b) Decide whether or not the graph in the figure is planar. Prove your answer. (Decription of hand-drawn figure: The graph has six vertices arranged in a regular hexagon. The edges are the sides and the main diagonals of the hexagon; so the graph is regular of degree three.)

2. (4 points) Let $\{b_n\}$ be a sequence of positive numbers such that $b_{n+1} = O(b_n)$. Prove: $\ln(b_n) = O(n)$.

3. (3 points) Evaluate the trinomial coefficient $\binom{7}{2, 2, 3}$. Do not use a calculator; show all your work.

4. (5 points) Write down the Laplacian of $K_{1,3}$. The vertex of degree 3 should be the first vertex.

5. (8 points) Recall that the "Prüfer code" assigns a sequence $P(T) = (p_1, \ldots, p_{n-2})$ of numbers to every tree on the vertex set $\{1, \ldots, n\}$ and yields a bijective proof of Cayley's formula n^{n-2}. Given T, describe how to construct its Prüfer code. Do not prove its correctness.

6. (5 points) Prove: if G is a planar graph with n vertices then G has an independent set of size $\geq n/6$. (Hint: chromatic number.)

7. (5 points) Find a closed-form expression for the ordinary generating function of the sequence $a_n = 1/(n+1)$ $(n = 0, 1, \dots)$.

8. (5 points) True or false: $\pi(x) = \Omega(x^{0.9})$. Prove your answer.

9. (4 points) Draw a strongly connected digraph which has no directed Hamilton cycle. Make your digraph as small as possible.

10. (5 points) BONUS PROBLEM. Prove: if G is a planar graph with n vertices and the complement of G is also planar then G has at most 10 vertices.

11. (5 points) BONUS PROBLEM. Give a closed-form expression of the **exponential** generating function of the Fibonacci numbers.