CMSC-37110 Discrete Mathematics: First Quiz 10-06-2005

Name (print): ______________________________

Show all your work. Do not use book or notes. Do not use separate sheets, write your answers in the space provided after each question. You may use a pocket calculator for basic arithmetic only (no binomial coefficients, etc.). If you are not sure you understand a problem properly, **ask the instructor.** The BONUS PROBLEMS are undervalued, do not solve them until you solved the regular problems.

This quiz contributes 6% to your course grade.

1. (5 points) In a well-shuffled deck of n cards, numbered 1 through n, what is the probability that cards #1 and #2 come next to each other (in either order)? Your answer should be an extremely simple expression; make it as simple as possible. Prove your answer.

2. (1+6 points) We pick a random word (string) of length n over the alphabet $\{A, B, C, D, E\}$. (a) How large is the sample space? (b) What is the probability that each letter actually occurs in the word? Give a closed-form expression (no dot-dot-dots or summation signs). (It will not be a very simple expression; do your best.)

3. (6 points) What is the expected number of Kings in a poker-hand? Prove your answer. Make sure you give a clear definition of each of the random variables you introduce. 4 out of the 6 points go for the definition.

(OVER)

4. (5 points) If two events, A and B, are positively correlated, what can we say about their complements, $\overline{A}$ and $\overline{B}$? Prove your answer.

5. (6 points) As a reward for her good deeds, Lucrezia gets to play the following game. She flips n coins. If k come up Heads, she wins 2^k ducats. What is her expected win? Give a simple closed-form expression (no dot-dot-dots or summation signs).

6. (1+3 points) Alice flips n coins and obtains X heads and $n - X$ tails. Bob repeats the experiment and obtains Y heads and $n - Y$ tails. (a) What is the size of the sample space for the combined experiment? (b) (BONUS PROBLEM) What is the probability that $X = Y$? Find a very simple closed-form expression (no summation or dot-dot-dots). Prove your answer. If you use a result proved in class, clearly state but do not prove that result.

7. (2 points) (BONUS PROBLEM) Prove: the product of k consecutive integers is always divisible by $k!$.

8. (3 points) (BONUS PROBLEM) Let $S_n = \displaystyle\sum_{j=0}^{\lfloor n/3 \rfloor} \binom{n}{3j}$. Prove: $\left| S_n - \dfrac{2^n}{3} \right| \leq \dfrac{2}{3}$.