1. (5 points) In a well-shuffled deck of $n$ cards, numbered 1 through $n$, what is the probability that cards #1 and #2 come next to each other (in either order)? Your answer should be an extremely simple expression; make it as simple as possible. Prove your answer.

2. (1+6 points) We pick a random word (string) of length $n$ over the alphabet \{A,B,C,D,E\}. (a) How large is the sample space? (b) What is the probability that each letter actually occurs in the word? Give a closed-form expression (no dot-dot-dots or summation signs). (It will not be a very simple expression; do your best.)

3. (6 points) What is the expected number of Kings in a poker-hand? Prove your answer. Make sure you give a clear definition of each of the random variables you introduce. 4 out of the 6 points go for the definition.
4. **(5 points)** If two events, $A$ and $B$, are positively correlated, what can we say about their complements, $\overline{A}$ and $\overline{B}$? Prove your answer.

5. **(6 points)** As a reward for her good deeds, Lucrezia gets to play the following game. She flips $n$ coins. If $k$ come up Heads, she wins $2^k$ ducats. What is her expected win? Give a simple closed-form expression (no dot-dot-dots or summation signs).

6. **(1+3 points)** Alice flips $n$ coins and obtains $X$ heads and $n - X$ tails. Bob repeats the experiment and obtains $Y$ heads and $n - Y$ tails. (a) What is the size of the sample space for the combined experiment? (b) (BONUS PROBLEM) What is the probability that $X = Y$? Find a very simple closed-form expression (no summation or dot-dot-dots). Prove your answer. If you use a result proved in class, clearly state but do not prove that result.

7. **(2 points)** (BONUS PROBLEM) Prove: the product of $k$ consecutive integers is always divisible by $k!$.

8. **(3 points)** (BONUS PROBLEM) Let $S_n = \sum_{j=0}^{\lfloor n/3 \rfloor} \binom{n}{3j}$. Prove: $\left| S_n - \frac{2^n}{3} \right| \leq \frac{2}{3}$.