

Discrete Mathematics – CMSC-37110-1 Homework 5

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HOMEWORK. Please print your name on each sheet. Please try to make your solutions readable. Unless expressly stated otherwise, all solutions are due at the **beginning of the next class**.

Please write solutions to challenge problems on a separate sheet.

HANDOUTS Chapters 5 and 6.1 from instructor's "Discrete Mathematics" lecture notes (DM): "Counting" and "Graphs and Digraphs." "Basic Number Theory" (NT).

READING: DM, Chap 6.1 "Graph Theory terminology" and NT, p.1. Relations, especially **equivalence relations**, from Matoušek - Nešetřil text or college Discrete Math texts.

DO: (exercises you need to do but not hand in):

DO5.1 View the exercises of DM, Chap 5.1 ("Counting") as practice problems.

DO5.2 DM 6.1 practice problems: 6.1.1 - 6.1.10, 6.1.12 - 6.1.22, 6.1.25, 6.1.26., 6.1.28 - 6.1.32.

DO5.3 Prove: a shortest walk between two vertices of a graph is a path (has no repeated vertices).

DO5.4 Practice problem: prove that the binomial coefficients $\binom{n}{k}$, $k = 0, \dots, n$ increase until the middle and then decrease.

Homework (due at the beginning of the next class):

HW5.1 (3+2+2+5) Count the isomorphisms between two copies of (a) C_n ; (b) P_n ; (c) K_n ; (d) $K_{r,s}$.

HW5.2 (3+3) What is the diameter of (a) C_n ($n \geq 3$); (b) $K_{r,s}$ ($r, s \geq 1$).

HW5.3 (8 points) Draw all non-isomorphic trees on seven vertices. State the number of trees you got. Lose two points for each error (missing a tree or repeating a tree) until 8 points exhausted.

CHALLENGE PROBLEMS:

- C5.1 (a1..a5) Count the isomorphisms between two copies of each Platonic solid (tetrahedron, cube, octahedron, dodecahedron, icosahedron) viewed as graphs. (b) Count the isomorphisms between two copies of the Petersen graph (DM 6.1, p. 54).
- C5.2 Let $g(n)$ denote the number of non-isomorphic graphs on n vertices. Prove: $\log_2 g(n) \sim n^2/2$.
- C5.3 DM 6.1.11 (Mandel - Turán Theorem).