

Discrete Mathematics – CMSC-37110-1    Homework 3 – October 4,  
2005

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**HOMEWORK.** Please print your name on each sheet. Please try to make your solutions readable. Unless expressly stated otherwise, all solutions are due at the **beginning of the next class.**

Please write solutions to challenge problems on a separate sheet.

Handout: Finite Probability Spaces (FPS) ("Chapter 7" of instructor's lecture notes)

Reading: FPS Chapter 7.2 (pp. 68-70)

Do: (exercises you need to do but not hand in):

DO3.1 Prove: if the events  $A, B, C$  are independent then  $A$  and  $B \cup C$  are also independent. More generally,  $A$  and any Boolean combination of  $B$  and  $C$  are independent. (Boolean combinations of sets are all sets that can be obtained from the given sets using the Boolean operations  $\cup$ ,  $\cap$ , and complementation.)

DO3.2 Prove: if the events  $A_1, \dots, A_k$  are independent then for any choice of  $\epsilon_i = \pm 1$ , the events  $A_1^{\epsilon_1}, \dots, A_k^{\epsilon_k}$  are independent. Here  $A_i^1$  means  $A_i$  and  $A_i^{-1}$  means  $\overline{A_i}$  (the complement of  $A_i$ ).

DO3.3 Let  $A_1, \dots, A_k$  be independent events. Let us split the set  $\{1, \dots, k\}$  into the disjoint union  $\{1, \dots, k\} = R_1 \dot{\cup} \dots \dot{\cup} R_k$ . Let further  $B_i$  be a Boolean combination of the events  $\{A_j : j \in R_i\}$ . Prove: the events  $B_1, \dots, B_s$  are independent.

DO3.4 FPS Ex. 7.2.4 (expectation is between min and max).

DO3.5 FPS Ex. 7.2.6 (linearity of expectation).

Homework (due at the beginning of the next class):

- HW3.1 (4 points) FPS Ex. 7.1.25.
- HW3.2 (4 points) Calculate the number of Boolean combinations of  $k$  events. (Note: in class we discussed that for  $k = 2$  this number was 16.) Prove your answer.

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- HW3.3 (6 points) FPS Ex. 7.2.11. Make sure you give an accurate definition of each random variable you introduce. The accuracy of these definitions accounts for 4 out of the 6 points.
- HW3.4 (8 points) FPS Ex. 7.2.13. Make sure you give an accurate definition of each random variable you introduce. The accuracy of these definitions accounts for 6 out of the 8 points.