Discrete Mathematics – CMSC-37110-1 Homework 2 – September 29, 2005

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HOMEWORK. Please print your name on each sheet. Please try to make your solutions readable. Unless expressly stated otherwise, all solutions are due at the **beginning of the next class.**
Handout: Finite Probability Spaces (FPS) ("Chapter 7" of instructor's lecture notes)

Reading: FPS Chapter 7.1 (pp. 63-67)
Review: concept of limits (calculus)
Do: (exercises you need to do but not hand in):

DO2.1 **Modular Identity.** Prove: if $A, B \subseteq \Omega$ are

$$P(A \cup B) + P(A \cap B) = P(A) + P(B).$$

DO2.2 **Union Bound.** Prove: if $A_1, \ldots, A_k \subseteq \Omega$ then

$$P(\bigcup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i).$$

Homework (due at the beginning of the next class):

- HW2.1 We deal 5 cards out of the standard deck of 52 cards. What is the probability that no two cards in the hand are the same kind (no two Kings, no two 9s, etc)? (a) (4 points) Give a *simple* formula. (b) (1 point) Write the answer as a quotient reduced to smallest terms (numerator and denominator relatively prime), showing the prime factors of the numerator and the denominator. (c) (1 point) Calculate your answer to 4 significant digits of accuracy.

- HW2.2 (Truncated Inclusion-Exclusion) Consider the events $A_1, \ldots, A_k \subseteq \Omega$; let $B$ be the complement of $\bigcup_{i=1}^{k}$. Then the Inclusion-Exclusion formula tells us that $P(B) = \sum_{i=0}^{k}(-1)^i S_i$. Prove:

[1] Changed "standard deck of $n$ cards" to "standard deck of 52 cards" in problem HW2.1. - L. B.

$$P(B) \leq S_0 - S_1 + S_2. \tag{1}$$

Challenge Problem. Generalize inequality 1. Prove that

$$P(B) \leq \sum_{i=0}^{\ell} (-1)^i S_i \tag{2}$$

if $\ell$ is even and

$$P(B) \geq \sum_{i=0}^{\ell} (-1)^i S_i \tag{3}$$

if $\ell$ is odd.