Algorithms – CS-27200/37000   Homework – February 11, 2004

Instructor: László Babai   Ry-164   e-mail: laci@cs.uchicago.edu

ADVICE. Take advantage of the TA's office hours Monday, Tuesday and Thursday 5–6pm in the Theory lounge (Ry–162).

DATES TO REMEMBER. Mon Feb 23: Quiz 2; Mon Mar 8: Midterm 2.

13.1 (U,G) (2+4+3 points) The *height* $h(x)$ of a node x in a rooted tree is the length of the longest path leading from x to a leaf. (Leaves have height zero.) The *leftheight* $lh(x)$ of a node in a binary tree is $1 + h(L(x))$ where $L(x)$ denotes the left child of x. If x has no left child ($L(x) =$NIL) then $lh(x) = 0$. The *rightheight* $rh(x)$ is defined similarly.

An *AVL-tree* is a binary tree in which every node x satisfies the inequality $|\, lh(x) - rh(x)| \le 1$. (The tree is "almost balanced" at every node.)

Let $m(t)$ denote the minimum number of nodes of an AVL-tree of height t. (The height of a rooted tree is the height of its root.) So $m(0) = 1, m(1) = 2, m(2) = 4$. (a) List $m(t)$ for $t \le 5$ and draw the corresponding minimal AVL-trees. (b) Find a simple recurrence satisfied by the sequence $m(t)$. (c) Determine the value $m(t)$ for all t. Your answer should be a very simple expression in terms of Fibonacci numbers. Prove your answers.