1. (6 points) Write a pseudocode for binary search. Do NOT use recursive calls. You need to search for a real number \( x \) in a sorted array \( A \) of \( n \) reals (\( A[1] \leq A[2] \leq \ldots \leq A[n] \)). Don’t forget proper rounding. Write the pseudocode only; do not analyze.

2. (3 points) Give a formal definition of the Knapsack problem. Use mathematical expressions with very few English words; those words should be technical terms (e.g. input, real variable, constraint, etc.) (do NOT use words like weight, value, knapsack, etc.).

3. (3 points) For two sequences of real numbers, \( \{a_n\} \) and \( \{b_n\} \), define the relation \( a_n \gtrsim b_n \) ("\( a_n \) is greater than or asymptotically equal to \( b_n \)").

4. (3 points) Prove: the depth of a sorting network which sorts \( n \) items must be \( \gtrsim 2 \log n \). (log stands for \( \log_2 \))

5. (G only, 2 points) Prove: \( \log(n!) \sim n \log n \).
6. A divide-and-conquer algorithm reduces an instance (=input) of size \( n \) to 3 instances of size \( n/2 \). (Ignore rounding.) The cost of the reduction is \( O(n) \). Let \( T(n) \) be the complexity of the algorithm on inputs of size \( n \).

(a) (1 point) Write a recurrence inequality for \( T(n) \).

(b) (1 point) State the problem (input-output) solved in class by a
an algorithm with the stated parameters. State the name of the
discoverers of the algorithm.

(c) (4 points) Ignore the cost of the reduction. Assuming \( n = 2^k \),
prove that \( T(n) = O(n^\alpha) \) where \( \alpha = \log 3 \).

(d) (G only, 4 points) Do not ignore the cost of the reduction. Prove
that \( T(n) = O(n^\alpha) \) where \( \alpha = \log 3 \). Use the method of reverse
inequalities.

7. (a) (5 points) Let \( X \) be an \( n \)-digit integer (decimal). We decide primal-
ity of \( X \) (i.e., whether or not \( X \) is a prime number) by trial division:
we divide \( X \) by each integer up to \( \lfloor \sqrt{X} \rfloor \). Is this a polynomial-time
algorithm? Prove your answer. (b) (G, only, 3 points) What if we only
divide by the prime numbers (up to \( \sqrt{X} \))?

8. (G only, 4 points) In the communication algorithm discussed in class,
suppose \( X = 213 \) and \( Y = 268 \). Suppose Alice chooses her prime
number \( p \) at random from all primes with at most two decimal digits.
What is the probability that Bob will erroneously report “\( X \) = \( Y \)”?
(Use the fact that \( \pi(100) = 25 \) where \( \pi(x) \) denotes the number of
primes \( \leq x \).) State what choices of \( p \) will cause this error.