The magical rabbit colony

Long ago, there was a colony of magical rabbits. These rabbits would reproduce at an abominable rate. Each rabbit would take a month to mature. After the initial month, the rabbit produces one offspring every month. Also these magical rabbits never die. The colony started with one lone rabbit.

Let $F_n$ denote the number of rabbits in the colony after $n$ months.

1. **(2 pts)** Calculate $F_1, F_2, F_3, F_4$ and $F_5$.

2. **(3 pts)** Prove that for $n > 2$, $F_n = F_{n-1} + F_{n-2}$.

3. **(5 pts)** Write a simple scheme function `fib` which takes an input a number $n$ and returns the value of $F_n$. Evaluate `(fib 30)` and notice how long it takes to calculate `(fib 30)`. If you are really adventurous you may try calculating `(fib 50)`.

   **Comments:** `fib2` takes about $n$ recursive calls to calculate $F_n$. One can actually write a `fib3` which requires less than $4 \log_2 n$ recursive calls to calculate $F_n$, using another clever trick.

4. **(5 pts)** Observe that the number of recursive calls needed to calculate `(fib 30)` is `(fib 30)`. Write another scheme function `fib2` to calculate $F_n$ so that the number of recursive calls to calculate `(fib2 30)` is only 30, and evaluate `(log (fib2 1476))`.

   **Hint:** calculate the pair $(F_{n+1}, F_n)$ from $(F_n, F_{n-1})$.

**Statistics**

Let $a_1, \ldots, a_n$ be a sequence of numbers. The mean (denoted $\mu$) and the Standard Deviation (denoted $\sigma$) are defined through the following formulae:

$$
\mu := \frac{(a_1 + a_2 + \ldots + a_n)}{n}
$$

$$
\sigma := \sqrt{\frac{(a_1 - \mu)^2 + (a_2 - \mu)^2 + \ldots + (a_n - \mu)^2}{n}}
$$

5. **(10 pts)** Write scheme function `mean` and `stddev` which consume lists of numbers and produce the mean and standard deviation of the input list respectively.