1. **Instruction selection**

   (a) Multiplication is much more expensive than other arithmetic operations such as addition or bit-shifts. Therefore, when multiplying by a constant it is often beneficial to use a sequence of such cheaper operations in place of a single multiply. The simplest example for this is multiplication by a power of 2 which can be implemented directly by a bit-shift. A slightly more complicated example is multiplication by 3 which only requires 2 cheap operations, recognizing that $x \times 3 = (x << 1) + x$:

   
   
   \[
   \begin{align*}
   t_1 & \leftarrow x \\
   t_2 & \leftarrow t_1 << 1 \\
   \ldots & \leftarrow t_1 + t_2 
   \end{align*}
   \]

   i. Show how to multiply by 10 using only 3 cheap operations.

   ii. Show how to multiply by 7 using only 4 cheap operations.

   iii. Can you reduce the number of cheap operations needed for $\times 7$ to just 2? How?

   (b) Generalizing your answers to questions 1(a)i and 1(a)iii above, outline an algorithm that given an arbitrary positive constant $c$ tries to minimize the number of cheap operations that implements multiplication by $c$. (Note: A general and optimal solution is a very difficult question. I am not looking for that. Just give it a good shot so that you can describe the idea in two paragraphs or so.)

2. **Flow analysis**

   (a) Do exercises 16.1, 16.2, and 16.3 from the textbook.

   (b) A definition $d : t \leftarrow \ldots$ of a temporary $t$ is *useless* if all the uses of $t$ that $d$ reaches are within other useless definitions.

   Describe the iterative, *work-list*-based dataflow analysis for finding all useless definitions of a program. (In particular, explain the following details: Which set is being computed iteratively? How is this set initialized? What do you keep on the worklist? When and how is the set updated? Why does your algorithm terminate?) You may assume that the results from an earlier reaching-definition analysis are available.