Homework assignment

CMSC 15300

Due: May 28, 2004

Recall the definition of substitution for the λ-calculus:

$$\begin{array}{rcll}
x[x \mapsto e'] & = & e' & \\
y[x \mapsto e'] & = & y & x \neq y \\
(e_1\ e_2)[x \mapsto e'] & = & e_1[x \mapsto e']\ e_2[x \mapsto e'] & \\
(\lambda x.e'')[x \mapsto e'] & = & \lambda x.e'' & \\
(\lambda y.e'')[x \mapsto e'] & = & \lambda y.(e''[x \mapsto e']) & x \neq y \text{ and } y \notin \mathrm{FV}(e') \\
(\lambda y.e'')[x \mapsto e'] & = & \lambda z.(e''[y \mapsto z][x \mapsto e']) & x \neq y,\ y \in \mathrm{FV}(e'), \text{ and } z \text{ is fresh}
\end{array}$$

The proof of the substitution lemma for the simply-typed λ-calculus presented in class is by induction on the definition of substitution. Define a well-founded partial ordering $\prec$ on λ terms Λ, such that Λ is well-founded under this ordering and that this ordering makes the proof of substitution a well-founded induction.

Hint: you will need to prove a lemma that $e \not\prec e[x \mapsto y]$, for any term e and variables x and y.