

1 Homework 7

Due in class Wednesday October 20th

Solve the following problems from the lecture notes:

7.1.19

7.1.22

Solve the following problem from the text page 348:

Problem 8

(a) Consider all ways to write a positive integer n as an ordered sum of positive integers. For example, 3 can be written as $1 + 1 + 1$, $1 + 2$, $2 + 1$, 3. 4 can be written as $1 + 1 + 1 + 1$, $1 + 2 + 1$, $1 + 1 + 2$, $2 + 1 + 1$, $3 + 1$, $1 + 3$, $2 + 2$. Next consider the number of ways to write n as an ordered sum with exactly k terms. So, the number of ways to write 3 with 2 terms is 2. The number of ways to write 4 with 3 terms is 3.

Give a bijection between the ways to write n as an ordered sum with exactly k terms and subsets of size $k - 1$ of a set of size $n - 1$.

(b) Now we want to write any positive integer n as an ordered sum of non-negative integers. For example, we can write 3 as an ordered sum of 2 non-negative integers as $2 + 1$, $1 + 2$, $3 + 0$, $0 + 3$.

Give a bijection between the ways to write n as an ordered sum of non-negative integers with exactly k terms and subsets of size $k - 1$ of a set of size $n + k - 1$.