Homework 15

Due Friday December 3rd.

Problem 1

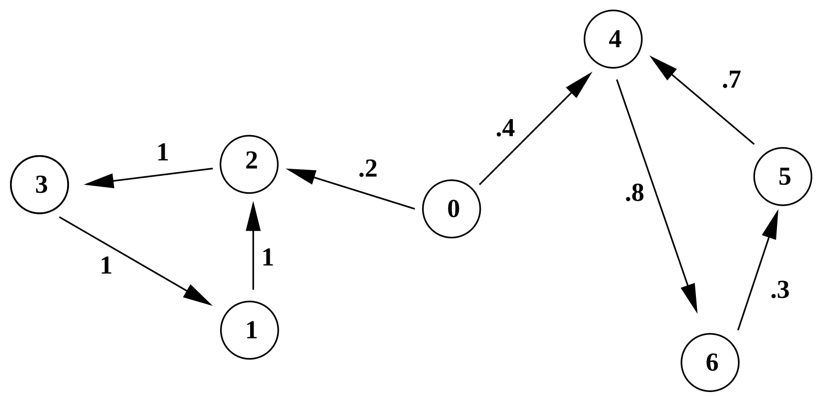

Figure 1: The Markov Chain for problem 1

Assume the Markov chain of Figure 1 is in state 0 just before the first step.
(a) What states are recurrent, transient, periodic?
(b) What is the probability that after 6 steps you are in state 3 ?
(c) What is the probability that after 6 steps you are in state 5 ?
(d) What is the expected number of steps before the process leaves state 0 ?
(e) What is the probability that we never get to state 6 ?

Problem 2
Say you have n homework problems and you have already solved k of them. Everyday you pick one problem equally at random to look at. If you haven't already, you solve the problem. Let X_i represent the event that i problems are unsolved.
(a) Prove that this process is a Markov chain with states X_i.
(b) What is the transition matrix for this chain?
(c) What states are recurrent, transient, periodic?

Problem 3
For the Markov chain of Figure 2:
(a) What are the recurrent classes of this Markov chain ?
(b) For each ergodic recurrent class determine the stationary distribution.
(c) What is the probability we eventually hit state 5 starting in state 0?
(d) What is the expected number of steps until we hit state 5?

Problem 4 For any graph, we can form a Markov chain by letting the

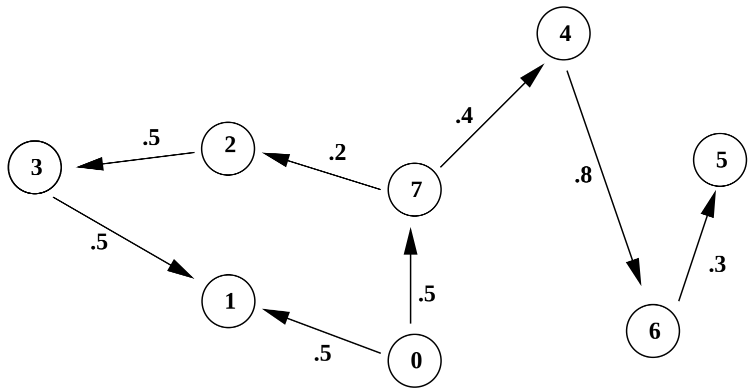

Figure 2: Markov chain for problem 3

transition from vertex v to any of its neighbors be $1/deg(v)$. Prove that such a Markov chain is periodic if and only if the graph is bipartite.

Problem 5

Suppose we have a Markov chain with two states and transition matrix:

$$\mathbf{T} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

(a) What conditions do we need to put on the p_{ij} in order for this chain to be ergodic?
(b) Under the conditions of part (a), find the stationary distribution of this chain.
(c) Suppose we want $\pi_1 = \pi_2$, then what values must the p_{ij} have?