Homework 13

Due in class Wednesday November 17th

1. If the generating function of the sequence $\{b_n\}$ is $g(x)$, what is the generating function of the sequence (a) $c_n = nb_n$; (b) $d_n = n^2 b_n$? (c) What sequence corresponds to the generating function $f(x) := g(x^2)$?

2. Let $\{e_n\}$ be the sequence defined by the recurrence $e_n = 2e_{n-1} + 3e_{n-2}$ ($n \geq 2$) with initial values $e_0 = 0$ and $e_1 = 1$. Find the generating function of the sequence $\{e_n\}$. Your answer should be a simple closed-form expression.

3. Prove: a connected graph with n vertices and m edges has at least $m - n + 1$ distinct cycles. *Hint.* Spanning trees.

4. What is the expected number of Kings in a poker hand? (There are 4 Kings in the standard deck of 52 cards; a poker hand consists of 5 cards.) Prove your answer. Half the credit goes for a clear definition of the variables you introduce.

5. We pay X dollars to enter the following game. We flip n coins. (n is given.) If k of the coins come up Heads, we receive 2^k dollars. What is the fair value of X? (Fainess means our expected gain should be zero.) Give a simple closed-form expression. Show all your work.