Homework 13

Due in class Wednesday November 17th

1. If the generating function of the sequence \( \{b_n\} \) is \( g(x) \), what is the generating function of the sequence (a) \( c_n = nb_n \); (b) \( d_n = n^2b_n \)? (c) What sequence corresponds to the generating function \( f(x) := g(x^2) \)?

2. Let \( \{e_n\} \) be the sequence defined by the recurrence \( e_n = 2e_{n-1} + 3e_{n-2} \) \((n \geq 2)\) with initial values \( e_0 = 0 \) and \( e_1 = 1 \). Find the generating function of the sequence \( \{e_n\} \). Your answer should be a simple closed-form expression.

3. Prove: a connected graph with \( n \) vertices and \( m \) edges has at least \( m - n + 1 \) distinct cycles. *Hint.* Spanning trees.

4. What is the expected number of Kings in a poker hand? (There are 4 Kings in the standard deck of 52 cards; a poker hand consists of 5 cards.) Prove your answer. Half the credit goes for a clear definition of the variables you introduce.

5. We pay \( X \) dollars to enter the following game. We flip \( n \) coins. \((n\text{ is given.})\) If \( k \) of the coins come up Heads, we receive \( 2^k \) dollars. What is the fair value of \( X \)? (Fainess means our expected gain should be zero.) Give a simple closed-form expression. Show all your work.