

Homework 12

Due: In class on Monday November 15th

(1) Let G be a random graph on n vertices (each edge is in G with probability $\frac{1}{2}$). What is the expected number of cycles of length k ? (Recall that two cycles on the same set of vertices with the same edges count as the same cycle.)

(2) n people are sitting at a round table. Everyone orders a different meal. The waiter puts the correct meal in front of each person. Then we spin the table such that any given plate stops equally at random in front of any given person. What is the expected number of people who have the correct meal in front of them?

(3) Let X be a random variable and α and β be constants.

Prove: $\text{Var}(\alpha X + \beta) = \alpha^2 \text{Var}(X)$.

(4) Let X be the number of letters which are put in the correct envelope from Exercise 7.2.16.

(a) What is the $\text{Var}(X)$?

(b) Prove that $p(X \geq 11) \leq .01$.

(5) Construct an example that shows the $\text{Var}(X + Y) \neq \text{Var}(X) + \text{Var}(Y)$ if X and Y are not independent.

(6) Let $a_n, b_n \rightarrow \infty$. Prove that if $a_n = \Theta(b_n)$ then $\ln a_n \sim \ln b_n$.

Solve the following problem from the Lecture Notes:

Exercise 2.7.8