CMSC 23700	Introduction to Computer Graphics	Homework 2
Fall 2004		Due October 19

1. (a) Construct the 4×4 matrix that corresponds to a 135 degree counter-clockwise rotation around the $X - axis$ followed by a translation by $\langle 1, 0, 0 \rangle$.

   (b) Given a unit cube with corners $\langle 0, 0, 0 \rangle$ and $\langle 1, 1, 1 \rangle$, what are its corners after transformation by the above matrix (give all eight coordinates)?

2. Affine transformations can be represented by 4×4 homogeneous matrices with the following shape:

$$\begin{bmatrix} \mathbf{M} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$$

   where $\mathbf{M}$ is a 3×3 matrix and $\mathbf{t}$ is a vector. We can use $\langle \mathbf{M} \mid \mathbf{t} \rangle$ as a more compact notation for this class of matrices. The product of two homogeneous matrices is

$$\langle \mathbf{M}_1 \mid \mathbf{t}_1 \rangle \langle \mathbf{M}_2 \mid \mathbf{t}_2 \rangle = \langle \mathbf{M}_1 \mathbf{M}_2 \mid \mathbf{M}_1 \mathbf{t}_2 + \mathbf{t}_1 \rangle$$

   and applying the transformation to a homogeneous point is

$$\langle \mathbf{M} \mid \mathbf{t} \rangle \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix} = \mathbf{M}_1 \mathbf{v} + \mathbf{t}$$

   If we restrict ourselves to isotropic (uniform) scaling, rotation, and translation, then these matrices are called *SRT* transforms and have the form $\langle s\mathbf{R} \mid \mathbf{t} \rangle$, where s is a scalar and $\mathbf{R}$ is a rotation matrix. Given this notation, solve the following equations:

   (a) $\langle s_1 \mathbf{R}_1 \mid \mathbf{t}_1 \rangle \langle s_2 \mathbf{R}_2 \mid \mathbf{t}_2 \rangle$

   (b) $\langle s\mathbf{R} \mid \mathbf{t} \rangle \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}$

   (c) $\langle s\mathbf{R} \mid \mathbf{t} \rangle^{-1}$

3. Given a ray $R(t) = \mathbf{o} + t\mathbf{d}$, and a cone whose radius is r and height is h with its base centered at the origin of the $X - Y$ plane and its apex at $\langle 0, 0, h \rangle$, what is the polynomial whose roots determine the intersection points of $R(t)$ with the cone?