CS Theory

• Computer Science = 
  applied mathematics + engineering

• CS theory is the applied mathematics part

• much of this concerns formalisms for computation (e.g. models of computation, programming languages) and their metatheory
Lesson 0: Course Introduction

Theory: Computability, Complexity

- computability theory
  - models of computation
  - what is computable, what is not
- complexity theory and analysis of algorithms
  - how hard or costly is it to compute something
  - what is feasibly computable
  - applications: design of efficient algorithms

Theory of programming

- semantics of computation
  - what do terms in a formalism mean?
- logics of computation (programming logics)
  - specifying computational tasks and verifying that programs satisfy their specifications
- computational logic
  - systems for automatic/interactive deduction
- type theory and type systems
  - which programs “make sense”
**What are type systems?**

“A type system is a *tractable syntactic* method for proving the absence of *certain program behaviors* by classifying phrases according to the kinds of values they compute.”

“A type system can be regarded as calculating a kind of *static* approximation to the run-time behaviors of the terms in a program.”

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**Thesis: Static typing is fundamental**

Static typing, based on a sound type system (“well-typed programs do not go wrong”) is a basic requirement for robust systems programming.
Why Types are Useful

- **error detection**: early detection of common programming errors
- **safety**: well typed programs do not go wrong
- **design**: types provide a language and discipline for design of data structures and program interfaces
- **abstraction**: types enforce language and programmer abstractions

Why Types are Useful (cont)

- **verification**: properties and invariants expressed in types are verified by the compiler ("a priori guarantee of correctness")
- **software evolution**: support for orderly evolution of software
  - consequences of changes can be traced
- **documentation**: types express programmer assumptions and are verified by compiler
Some history

- 1870s: formal logic (Frege), set theory (Cantor)
- 1910s: ramified types (Whitehead and Russell)
- 1930s: untyped lambda calculus (Church)
- 1940s: simply typed lambda calc. (Church)
- 1960s: Automath (de Bruijn); Curry-Howard correspondence; Curry-Hindley type inference; Lisp, Simula, ISWIM
- 1970s: Martin-Löf type theory; System F (Girard); polymorphic lambda calc. (Reynolds); polymorphic type inference (Milner), ML, CLU

Some History (cont)

- 1980s: NuPRL, Calculus of Constructions, ELF, linear logic; subtyping (Reynolds, Cardelli, Mitchell), bounded quantification; dependent types, modules (Burstall, Lampson, MacQueen)
- 1990s: higher-order subtyping, OO type systems, object calculi; typed intermediate languages, typed assembly languages
Lesson 0: Course Introduction

**Course Overview**

- Part I: untyped systems
  - abstract syntax
  - inductive definitions and proofs
  - operational semantics
  - inference rules

- Part II: simply typed lambda calculus
  - types and typing rules
  - basic constructs: products, sums, functions, ...
  - intro to type safety

**Course Overview (cont)**

- Part III: subtyping
  - metatheory
  - case studies (imperative objects)

- Part IV: recursive types
  - iso-recursive and equi-recursive forms
  - metatheory (coinduction)

- Part V: polymorphism
  - ML-style type reconstruction
  - System F
  - polymorphism and subtyping: bounded quantifiers
Course Overview (cont)

- Part VI: Type operators
  - higher-order type constructs
  - System $F_{\omega}$
  - subtyping: System $F_{\omega}^<$
  - case study: functional objects

Topics omitted

- type systems as logics
- denotational semantics of programs and types
- module systems
- full-featured object-oriented languages
Required background

The course is self-contained, but the following will be useful:

- "mathematical maturity"
- some familiarity with (naive) set theory, elementary logic, induction
- some familiarity with a higher-order functional language (e.g. scheme or ML or Haskell)

Implementation

- Several chapters present implementations of type checkers.
- The programming language used in the text is a simple subset of Ocaml. In the course, I will substitute code in a similar subset of Standard ML.
- For documentation/tutorials on Standard ML, see www.smlnj.org