Problem 1: Queues

Consider the queue implementation discussed in class in which a queue is represented by three variables, head, tail, queue, all of which are defined at the top-level. head and tail are integers, and queue is (vectorof number). Assume the basic, non-circular vector implementation of queues.

Let’s design a function print-queue that non-destructively prints all the elements in the queue in order of their position in the queue.

Examples

Create an example queue to print using the procedures enqueue and dequeue. There should be at least 4 entries in the queue. You must include calls to both enqueue and dequeue.

Give the sequence of enqueue and dequeue calls.

Solution

(enqueue 1)
(enqueue 2)
(enqueue 3)
(dequeue)
(dequeue)
(enqueue 4)
(enqueue 5)
(enqueue 6)

Function

Create a function called print-queue that builds a list of the numbers in the queue.

(define (print-queue)
  ;; print-queue: -> (listof numbers)

  Solution

  (define (print-queue)
    (print-queue-aux head))
(define (print-queue-aux current)
  (if (>= current tail)
      '()
      (cons (vector-ref current queue)
            (print-queue-aux (+ current 1)))))

Test
Show the output of your function on the example queue you constructed in part one.
Solution
(3 4 5 6)

Alternate implementations
We also considered a circular implementation of queues using vectors in class.

Would the function print-queue change under the circular vector implementation of queues? If so, please modify your function to handle the circular queue implementation. If not, explain why.
Solution Yes, it would need to change because we could no longer assume head and tail were less than (vector-length queue).

(define (print-queue)
  (print-queue-aux head))

(define (print-queue-aux current)
  (if (>= current tail)
      '()
      (cons (vector-ref (modulo current (vector-length queue)) queue)
            (print-queue-aux (+ current 1)))))

Abstraction and Concrete Implementation
Explain what would happen if you used dequeue to implement print-queue.
Solution Since dequeue is a destructive operation - in other words it causes the side-effect of removing an element from the queue by changing
the position of head, you would not be able to run the destructive version of print-queue twice. The second time would report that the queue was empty.

You could, of course, circumvent this problem by re"enqueuing" all of the deleted elements after forming the list.

### Problem 2: Streams: Powers

#### Stream Creation

Recall the data definition of streams:

A stream is:

- (cons number (delay stream))

Create a stream powers-of-2 composed of the powers of two, i.e. $2^1, 2^2, ..., 2^i, ...$

There are many alternative ways to define a stream of powers-of-2. Feel free to create powers-of-2 in whatever way makes sense to you.

Here’s a function to help you test your code.

```scheme
;; take : number stream -> list-of-numbers
;; extracts the first n elements from the stream
(define (take n s)
  (cond
   [(zero? n) '()]  
   [else (cons (car s) (take (- n 1) (force (cdr s))))]))
```

**Hint:** One possibility is to create a function powers-of-2-from which takes a starting number n and creates a stream consisting of $n \times 2^0, n \times 2^1, n \times 2^2, ... , n \times 2^i, ...$

```scheme
(define (powers-of-2-from n)
  ;; powers-of-2-from: number -> stream-of-numbers
  (define (powers-of-2-from n)
    (cons n (delay (powers-of-2-from (+ 2 n)))))
```
(define powers-of-2 (powers-of-2-from 2))

Alternate solutions include: (define powers-of-2 (cons 2 (delay (map-stream (lambda (x) (* x 2)) +powers-of-2))))

Powers: Generalization

Consider a function power-stream that will construct a stream of all the powers of any x, starting with the first power; that is, a stream given by the sequence \(x, x^2, x^3, x^4, \ldots\) for ANY value of x.

(define (power-stream x)
  ;; power-stream: number -> stream-of-numbers

Examples

Give three examples of calls to power-stream and the first few elements of the output stream.

Solution

(power-stream 1) => (1 1 1 1 1 1 ...
(power-stream 2) => (2 4 8 16 32 ...
(power-stream 10) => (10 100 1000 10000 ...

Template

Write the template for functions on streams.

Solution

(define (fn-for-stream stream)
  ... (car stream) ...
  ... (fn-for-stream (force (cdr stream)))... .

Function

Create the function power-stream as described above.

Solution

(define (power-stream x)
  (cons x
    (delay (map-stream (lambda (z) (* x z)) (power-stream x)))))