Problem 1: (5 points) Assume that all possible Turing machines are written down in the sequence $M_1, M_2, M_3, \ldots$. Assume that all strings of finite length are written down without any repetitions in the sequence $w_1, w_2, w_3, \ldots$. Prove that the language

$$L_d = \{w_i | M_i \text{ does not accept } w_i\}$$

is not r.e. (This is Lemma 8.1 in the text and was discussed in class.)

Problem 2: Assume there is an algorithm to decide if two Turing machines $M_1$ and $M_2$ accept exactly the same language. Use that algorithm to construct

(a) (10 points) an algorithm to decide if the language accepted by a Turing machine $M$ is empty,

(b) (10 points) an algorithm to decide if a Turing machine $M$ accepts a string $w$ (this is $L_u$ in the text).

(5 points) Argue that no algorithm can tell if two C programs perform the same task in possibly different ways.

Problem 3: (10 points) Assume there is an algorithm to decide if a Turing machine $M$ accepts a string $w$ (this is $L_u$ in the text). Use this to give an algorithm to decide if a Turing machine $M$ halts on the input $w$ (this is the halting problem).