Homework 3 - Due Wednesday October 22nd

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

1. Give a closed form solution to the number of solutions in positive integers to the equation:

$$x_1 + x_2 + \ldots + x_{10} \leq 100$$

2. Let (x_i, y_i), $i = 1 \ldots 5$ be a set of five distinct points with integer coordinates in the xy plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.

3. Prove that if $f_1 = o(g_1)$, then $f_1 + g_1 \sim g_1$.

4. Find the asymptotic relations between f_n and g_n (O, o, Θ). (Note that more than one relation might hold.)

   (a) $f_n = 2^n$ $g_n = 2^{n/2}$

   (b) $f_n = log(n!)$ $g_n = log(n^n)$

5. An n-bit *boolean* function f maps a 0/1 string of length n to 0 or 1, $f : \{0,1\}^n \rightarrow \{0,1\}$. (a) Count the number of n-bit boolean functions. An n-bit boolean function *depends* on i if $\exists$ two strings A and B s.t. A and B differ only in position i and $f(A) \neq f(B)$. (b) Count the number of n-bit boolean functions that do **not** depend on bit i.

6. How many permutations (rearrangements) of the letters:

   M,A,T,H,I,S,F,U,N

   are there such that none of the words MATH, IS, and FUN occur as consecutive letters? (So for example, MATHISFUN, INUMATHSF, and ISMATHFUN, are all not allowed)