Discrete Math - Homework 6 - Due Wednesday November 19th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

1. Study for the midterm.

2. A box contains three coins. One has two heads, one is a fair coin, and the other comes up heads 70 percent of the time. A coin is selected equally at random from the box and tossed. If it comes up heads, what is the probability it was the fair coin?

3. What is the minimum number of people who need to be in a room so that the probability that at least two of them have the same birthday is at least $\frac{1}{2}$? (Make the following assumptions, birthdays are independent and equally likely to fall on any day of the year and a year has 365 days.)

4. Suppose a permutation of $[5]$ is choosen equally likely at random. What is the expected number of entries in the permutation that are greater than all preceeding entries? For example, in the permutation $3, 5, 2, 4, 1$, the numbers 3 and 5 are greater than all preceeding entries.

5. Prove that every graph has at least two vertices of the same degree.

6. Let $G = (V, E)$ be a graph with $|V| = n \geq 3$. Prove that G has a Hamiltonian cycle if for every pair of non-adjacent vertices, u and v, $\deg(u) + \deg(v) \geq n$.

7. In class we noted that every tree is bipartite. Prove that every tree has a leaf in its larger partite set (in both if they have equal size).