Discrete Math - Homework 4 - Due Wednesday November 12th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

1. A bus route connects downtown with the local university campus. Mr. X., a student at the university, takes the bus from downtown every weekday after he wakes up, which happens at a random time of the day (24 hours). According to his records, he has to wait for the bus for 30 minutes on average. At the same time, the bus company claims that the average interval between two buses during the day (over all 24 hours) is 15 minutes. Can you construct a schedule such that both Mr. X. and the bus company are right?

2. Define a random graph on n vertices to be a graph formed as follows: Fix a vertex set $[n]$, for each pair of vertices x, y let (x, y) be an edge with probability p (flip a biased coin). (a) What is the expected number of edges in a random graph? (b) What is the expected number of triangles in a random graph?

3. Give an example of two graphs that have the same number of vertices, the same number of edges, and the same degrees, but are not isomorphic. (Remember to prove your answer.)

4. Prove that a graph is bipartite iff it contains no odd cycles (as a subgraph).

5. Suppose we color the *edges* of K_6 with two colors. Prove that no matter how we color the edges there will exist a monochromatic triangle (A triangle of only one color).

6. Prove that given positive integers $d_1, \ldots, d_n$ that sum to $2n-2$, there are exactly $\frac{(n-2)!}{\prod(d_i-1)!}$ trees on n vertices such that vertex i has degree d_i.