Discrete Math - Homework 4 - Due Wednesday November 5th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

1. Suppose you receive equal amounts of spam e-mail and non-spam e-mail. Further suppose the probability that a spam e-mail contains the word "free" is $\frac{1}{3}$ and the probability that a non-spam e-mail contains the word "free" is $\frac{1}{30}$. You receive an e-mail that contains the word "free" what is the probability that the e-mail is spam?

2. Give an example of events X_1, X_2, and X_3 such that the events are pairwise independent but not independent as a triple.

3. Suppose a multiple choice problem has n possible answers. You guess an answer equally likely at random. If you get it wrong, you pick an answer equally at random from the remaining choices. You continue until you have guessed the right answer, each time only guessing from answers you have not tried yet.

   (a) What is the expected number of guesses until you are correct?

   (b) What is the variance of the number of guesses you take until you get the right answer?

4. Suppose you roll a fair 6 sided die 100 times. Let X be the number of times two consecutive rolls result in the same number. (a) What is the $E[X]$? (b) What is the $V(X)$?

5. Suppose in a lottery you have to pick five different numbers from 1 to 90. Then five winning numbers are drawn. If you picked two of them, you win 20 dollars. For three, you win 150 dollars. For four, you win 5,000 dollars, and if all five match, you win a million dollars. (a) What is the probability that you picked exactly three of the winning numbers? (b) What is your expected win? (c) What does Markov's inequality predict about the probability that you'll win at least 20 dollars? (d) What is the actual probability that this happens?

6. Let X_n be the number of occurances of the string "TEST" in a random string of length n over the english alphabet of 26 letters. ("TEST" needs to appear as 4 consecutive letters, like in DCCTESTGHA.) For what value of n is $E[X_n] = 1$?