Discrete Math - Homework 2 - Due Wednesday Oct. 15th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

1. Suppose we have a collection of subsets of $[n]$ such that each pair of subsets has at least one element in common. What is the maximum possible size of the collection?

2. How many functions $f : \{1, \ldots, n\} \to \{1, \ldots, n\}$ are there that are monotone; that is, for $i < j$ we have $f(i) \leq f(j)$?

3. Let X be a set of size n. A partition of X into k blocks is defined as:

   $$X = \{B_1, B_2, \ldots, B_k\}$$

   such that

   - For all i, $B_i \neq \emptyset$
   - For all $i \neq j$, $B_i \cap B_j = \emptyset$
   - $B_1 \cup B_2 \cup \ldots \cup B_k = X$.

   For all $n \geq k$ define: $S(n, k)$ to be the number of partitions of an n element set into k blocks. Note some base cases: $S(0, 0) = 1$, for $n \neq 0$ $S(n, 0) = 0$, $S(n, 1) = 1$.

   (a) Prove the following recurrance:

   $$S(n, k) = kS(n-1, k) + S(n-1, k-1)$$

   (b) Prove by giving a bijection that the number of onto maps from an n element set to a k element set is

   $$k!S(n, k)$$

4. Give a formula for the number of onto maps from an n element set to a k element set. (You will not get a closed form for this problem, but the expression is still quite nice.)

5. Prove whether or not the following sequences are polynomially bounded.
   (a) $n^3 \ln(n^2 + 5)$
   (b) $5^{\ln n}$