Discrete Math - Homework 1 - Due Wednesday Oct. 8th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

1. Give a combinatorial proof (i.e. show both sides count the same thing) of the following identity:

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$$

2. A composition of a positive integer n with k parts is an ordered sum $x_1+x_2+\cdots+x_k$ such that each x_i is a positive integer. Give a bijection between compositions of n with k parts and subsets of $[n-1]$ of size $k-1$.

3. Prove that if p is a prime number and $1 \leq k \leq p-1$ then p divides the binomial coefficient $\binom{p}{k}$.

4. Give a closed form expression for the number of ordered pairs (A, B), where $A \subseteq B \subseteq \{1, \ldots, n\}$.

5. Suppose we place any 5 points in a square that has side length equal to 2. Prove that two of the points must be within $\sqrt{2}$ distance of each other.