

Homework 8 - Due Wednesday November 28th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

1. Prove that powers of the adjacency matrix count paths in a graph. Namely, if A is the adjacency matrix for a directed graph G , prove that entry i, j of A^t is the number of directed walks from i to j of length t .
2. If T is the transition matrix for some Markov chain M , and q_t is the distribution of the random variable X_t recording the state after t steps, prove that $q_t = q_0 T^t$.
3. For any graph, we can form a Markov chain by letting the transition from vertex v to any of its neighbors be $1/\deg(v)$. Suppose we start at any state equally at random. Prove that such a Markov chain is periodic (i.e. all states are periodic) if and only if the graph is bipartite.