Homework 9 - Due Wednesday November 29th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

1. As defined in section 5.4 of your text, A coloring of a graph $G = (V, E)$ is a mapping $c : V \to [k]$ such that $c(x) \neq c(y)$ for every edge $\{x, y\} \in E$. The chromatic number of G, $\chi(G)$, is the minimum k such that there exists a coloring of G $c : V \to [k]$.

   The famous Four Color Theorem states that any planar graph has chromatic number at most 4!

   (a) Prove that in any planar graph there must exist a vertex of degree at most 5.

   (b) Prove the 6 color theorem: Any planar graph has chromatic number at most 6.

2. Prove that a graph is bipartite if and only if its chromatic number is at most 2.

3. Suppose we have a Markov chain with the following transition matrix:

$$\mathbf{T} = \begin{pmatrix} .5 & .5 & 0 & 0 & 0 & 0 & 0 \\ .5 & .4 & .1 & 0 & 0 & 0 & 0 \\ 0 & .6 & .4 & 0 & 0 & 0 & 0 \\ 0 & 0 & .2 & .4 & .2 & .2 & 0 \\ 0 & 0 & 0 & .7 & 0 & 0 & .3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

   (a) Which states are transient, recurrent, periodic?

   (b) Find the stationary distribution given that we started in state 1, or say why it doesn't exist.

   (c) Find the stationary distribution given that we started in state 6, or say why it doesn't exist.

   (d) Given that we start in state 4 what is the probability that we ever hit state 3?

4. Problem 3.2.9 part (a) of your text.

5. Problem 3.2.10 of your text.