Homework 8 - Due Wednesday November 22nd

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

1. Prove that a graph is bipartite if and only if it contains no odd cycle.
2. Does there exist a graph on 6 vertices which is (a) Hamiltonian but not Eulerian (b) Eulerian but not Hamiltonian?
3. Suppose we color the *edges* of K_6 with two colors. Prove that no matter how we color the edges there will exist a monochromatic triangle (A triangle of only one color).
4. Prove that G is a tree if and only if G is connected and has exactly $n-1$ edges.
5. Prove that given positive integers $d_1, \ldots, d_n$ that sum to $2n-2$, there are exactly $\frac{(n-2)!}{\prod(d_i-1)!}$ trees on n vertices such that vertex i has degree d_i.
6. Problem 5.3.3 from your text. Prove that a planar graph in which each vertex has degree at least 5 must have at least 12 vertices.