Homework 6 - Due Wednesday November 8th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

1. Suppose a permutation of [5] is choosen equally likely at random. What is the expected number of entries in the permutation that are greater than all preceeding entries? For example, in the permutation $3, 5, 2, 4, 1$, the numbers 3 and 5 are greater than all preceeding entries.

2. Suppose you have two coins: one is fair and the other comes up heads with probability $\frac{3}{4}$. One of the two coins is tossed n times. How large should n be to make us 95 percent confident which coin was chosen?

3. Let X_n be the number of occurances of the string "TEST" in a random string of length n over the english alphabet of 26 letters. ("TEST" needs to appear as 4 consecutive letters, like in DCCTESTGHA.) For what value of n is $E[X_n] = 1$?

4. We have a deck of n cards, numbered $\{1, 2, \ldots, n\}$. A hand consists of k cards. Suppose you are dealt a hand equally at random from all possible hands, what is the probability that a hand has no consecutive numbers?

5. Problem 9.2.6 from your text. Show that if A and B are independent events then also their complements, $\Omega \backslash A$ and $\Omega \backslash B$ are independent.

6. Problem 9.3.5 from your text. A bus route connects downtown Old Holstein with the local university campus. Mr. X., a student at the university, takes the bus from downtown every weekday after he wakes up, which happens at a random time of the day (24 hours). According to his records, he has to wait for the bus for 30 minutes on average. At the same time, the bus company claims that the average interval between two buses during the day (over all 24 hours) is 15 minutes. Can you construct a schedule such that both Mr. X. and the bus company are right?