Homework 5 - Due Wednesday November 1st

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

1. Suppose you roll a fair 6 sided die 100 times. Let X be the number of times two consecutive rolls result in the same number. (a) What is the $E[X]$? (b) What is the $V(X)$?

2. Prove that if X and Y are two independent random variables, then

$$E[XY] = E[X]E[Y]$$

3. Prove that if X and Y are two independent random variables, then

$$V(X+Y) = V(X) + V(Y)$$

and

$$V(aX+b) = a^2V(X)$$

where a and b are constants.

4. We roll a fair 6-sided die n times. What is the probability that k out of the n times prime numbers comes up?

5. Suppose we put k balls into n boxes, each ball gets put into a box equally at random. (a) What is the expected number of balls in any given box? (b) What is the expected number of empty boxes?

6. Suppose in a lottery you have to pick five different numbers from 1 to 90. Then five winning numbers are drawn. If you picked two of them, you win 20 dollars. For three, you win 150 dollars. For four, you win 5,000 dollars, and if all five match, you win a million dollars. (a) What is the probability that you picked exactly three of the winning numbers? (b) What is your expected win? (c) What does Markov's inequality predict about the probability that you'll win at least 20 dollars? (d) What is the actual probability that this happens?