

Homework 3 - Due Wednesday October 18th

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions

(1) Let O_n be the set of all odd subsets of $[n]$. Let E_n be the set of all even subsets of $[n]$. Give a bijection between O_n and E_n . Note: the empty set which has size 0 is considered an even subset.

(2) Prove that:

$$a_n \sim b_n \text{ and } c_n \sim d_n \Rightarrow a_n + c_n \sim b_n + d_n$$

is false in general. Hint: if $a_n c_n > 0$ then the statement is true.

(3) Prove whether or not the following sequences are polynomially bounded.

(a) $n^3 \ln(n^2 + 5)$

(b) $5^{\ln n}$

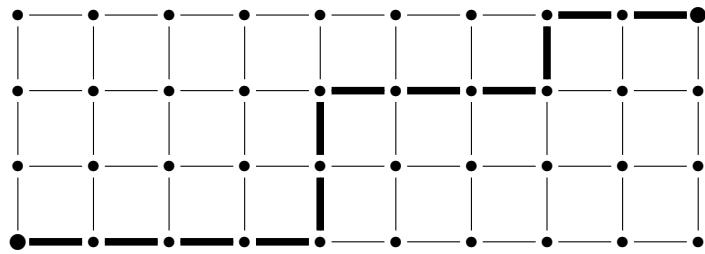
(4) Prove:

$$\binom{2n}{n} \sim \frac{4^n}{\sqrt{\pi n}}$$

(5) Problem 2.3.7 from your text. How many functions $f : [n] \rightarrow [n]$ are there that are monotone; that is, for $i < j$ we have $f(i) \leq f(j)$?

(6) Out of n candidates, an association elects a president, two vice presidents, and a treasurer. Count the number of possible outcomes of the election.

(7) Count the shortest paths from the bottom left corner to the top right corner of the $n \times k$ grid. (The figure shows a 4×10 grid with a shortest path in question highlighted.)



(Optional Challenge) (a) Let p_n be the n -th prime number. Prove, using the Prime Number Theorem, that $p_n \sim n \ln n$.